Mathematics

WORK SAMPLE PORTFOLIOS

These work sample portfolios have been designed to illustrate satisfactory achievement in the relevant aspects of the achievement standard.

The December 2011 work sample portfolios are a resource to support planning and implementation of the Foundation to Year 10 Australian Curriculum in English, Mathematics, Science and History during 2012. They comprise collections of different students' work annotated to highlight evidence of student learning of different aspects of the achievement standard.

The work samples vary in terms of how much time was available to complete the task or the degree of scaffolding provided by the teacher.

There is no pre-determined number of samples required in a portfolio nor are the work samples sequenced in any particular order. These initial work sample portfolios do not constitute a complete set of work samples - they provide evidence of most (but not necessarily all) aspects of the achievement standard.

As the Australian Curriculum in English, Mathematics, Science and History is implemented by schools in 2012, the work sample portfolios will be reviewed and enhanced by drawing on classroom practice and will reflect a more systematic collection of evidence from teaching and learning programs.

THIS PORTFOLIO – YEAR 10 MATHEMATICS

This portfolio comprises a number of work samples drawn from a range of assessment tasks, namely:

Sample 1	Maths assignment
Sample 2	Number plane graphs
Sample 3	Linear equations – Taxi fares
Sample 4	Similar or congruent triangles
Sample 5	Trigonometry assignment
Sample 6	Algebraic expressions
Sample 7	Algebraic fractions

This portfolio of student work shows the completion of tables of values and the ability to draw graphs of linear and non-linear relationships and explain the features of the graph (WS3). They make the connections between algebraic and graphical representation of relations (WS1, WS2, WS3). The student collects data from an experiment, uses mathematical knowledge and skills to investigate the data and uses reasoning to draw conclusions (WS4, WS7). The student identifies similar and congruent triangles and demonstrates understanding of similarity and congruence (WS4). The student solves simple algebraic fractions with the four operations (WS7) and expands binomial expressions (WS6). The student understands basic right-angled triangle trigonometry (WS5), labeling sides of a triangle and writing the ratios for a given triangle. The student applies knowledge to problems involving angles of elevation and depression and three figure bearings (WS5). The student describes bivariate data where the independent variable is time and the relationships between two continuous variables and evaluates statistical reports (WS5).

Mathematics

The following aspect/s of the achievement standard is not evident in this portfolio:

- *recognise the connection between simple and compound interest*
- *solve surface area and volume problems relating to composite solids*
- *recognise the relationships between parallel and perpendicular lines*
- *solve pairs of simultaneous equations*
- *use trigonometry to calculate unknown angles in right-angled triangles*
- *list outcomes for multi-step chance experiments and assign probabilities for these experiments*
- *calculate quartiles and inter-quartile ranges.*

Mathematics

Work sample 1:
Maths assignment

Relevant parts of the achievement standard

By the end of Year 10, students recognise the connection between simple and compound interest. They solve problems involving linear equations and inequalities. They make the connections between algebraic and graphical representations of relations. Students solve surface area and volume problems relating to composite solids. They recognise the relationships between parallel and perpendicular lines. Students apply deductive reasoning to proofs and numerical exercises involving plane shapes. They compare data sets by referring to the shapes of the various data displays. They describe bivariate data where the independent variable is time. Students describe statistical relationships between two continuous variables. They evaluate statistical reports.

Students expand binomial expressions and factorise monic quadratic expressions. They find unknown values after substitution into formulas. They perform the four operations with simple algebraic fractions. Students solve simple quadratic equations and pairs of simultaneous equations. They use triangle and angle properties to prove congruence and similarity. Students use trigonometry to calculate unknown angles in right-angled triangles. Students list outcomes for multi-step chance experiments and assign probabilities for these experiments. They calculate quartiles and inter-quartile ranges.

Summary of task

This group assignment was completed at the end of a semester. It assessed several topics including quadratic equations, bivariate data, statistics and algebraic graphical representations.

In this assignment students collected data from an experiment. The assignment measured the student's understanding of the interrelationships of mathematical concepts and reasoning to draw conclusions based on the data. The students were given one week to complete the task.

Mathematics

Work sample 1:
Maths assignment

Maths assignment- year 10

Part A

Knowledge and procedures

- ✓ Task 1: Create models of the whirlybird
- ✓ Task 2: Test fly whirlybirds detailing the conditions
- ✓ Task 3: Collect, record and summarise the data obtained in an appropriate form
- ✓ Task 4: Produce a scatter plot of the data

Task 1 was completed successfully as a team collaboration. Layouts and designs of the whirlybirds were provided to each student and the diagrams in Appendix 1 were followed to complete the models of the whirlybirds. Since the time a whirlybird is in flight may be effected by the length of the wings, the width of the wings and/or the size of its body tests were conducted to see if variations in length had any effect on how long the whirlybird was in flight for. Measurements were taken of the length of the whirlybird's wing and it measured to be 15.3cm in length therefore 3mm were cut off to make the measurement a round number. The length of the eight different whirlybird wing differed in sizes of two cm. Starting as small as a length of one cm and increasing to 15 cm. Once the layouts of the whirlybirds were cut out a paper clip was attached to the bottom of the folded whirlybird to keep the paper intact.

Task 2, the testing of the whirlybirds, was conducted in the same conditions and we the same variables were current in each test. The variables that were constant are listed below:

- The width of the wings on the whirlybird were the same
- The size of the whirlybirds body's were the same
- The same timer (stopwatch) was used
- The whirlybirds were thrown from the same height (220.5cm)
- The whirlybirds were thrown by the same person using the same technique every time
- The whirlybirds were in the same controlled environment (inside the classroom)
- The same person timed every whirlybird flight through the course of the experiment

Unfortunately, some variables may have affected by the time that the whirlybird was in flight for and these changing variables are listed below:

- The wing lengths of the whirlybirds were purposely changed to see if it had any effect on the time the whirlybirds were in flight for
- One side of the windows were open in the classroom and the conditions could most probably have changed throughout the course of the experiment. Since the speed of the wind isn't always constant
- Louise (in charge of throwing the whirlybirds) could have accidently applied more strength in her releases/throws for some of the whirlybirds. This would ultimately effect the time the whirlybird were in flight for

Tests to see how long the whirlybirds were in flight for were conducted three times per measurement which meant that an average result of how long the whirlybirds were in flight for

Annotations

Demonstrates the ability to design investigations and plan their approach to answering a question.

Interrogates their solutions and uses a number of different approaches to subject them to rigorous scrutiny, successfully verifying that their answers are reasonable.

Chooses appropriate methods and approximations.

Identifies a variety of variables that impact upon the results of trials.

Presents meticulous records of trials and working.

Mathematics

Work sample 1:
Maths assignment

could be found. By finding an average in the results we could eliminate the possibility that the changing variables could have any effect on the flight time of each whirlybird.

The table below is a collection of the data obtained during the experiment. The first column is the measurements in centimetres of the whirlybirds changing wing lengths. The next columns represent the three tests that were conducted. The total represents the addition of the three tests for each measurement of wing length. The average is found by adding the flight times from the three different tests and dividing that number by the number of tests conducted, which in this case was three. The table below shows the results from the experiment.

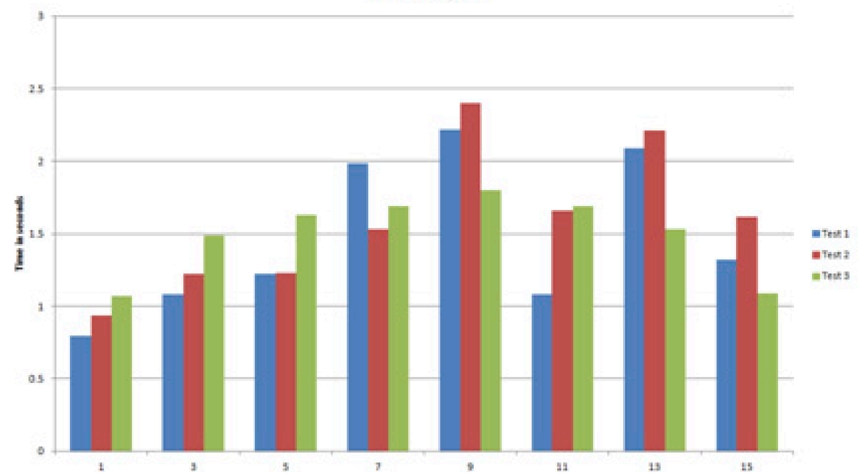
(Figure 1)

Cm	Test 1	Test 2	Test 3	Total	Average
1	0.8	0.94	1.07	2.81	0.936667
3	1.08	1.22	1.49	3.79	1.263333
5	1.22	1.23	1.63	4.08	1.36
7	1.99	1.53	1.69	5.21	1.736667
9	2.22	2.4	1.8	6.42	2.14
11	1.08	1.66	1.69	4.43	1.476667
13	2.09	2.21	1.53	5.83	1.943333
15	1.32	1.62	1.09	4.03	1.343333

The column graph below aims at demonstrating furthermore how the length of the wing of the whirlybirds could effect the results.

(Figure 2)

Test to determine which length of wing produces the longest flight time for the whirlybird



Annotations

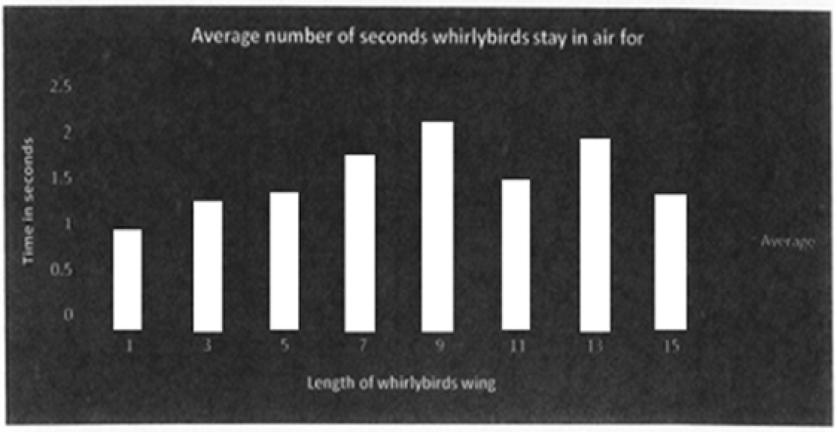
Collects, records and graphs data.

Determines averages.

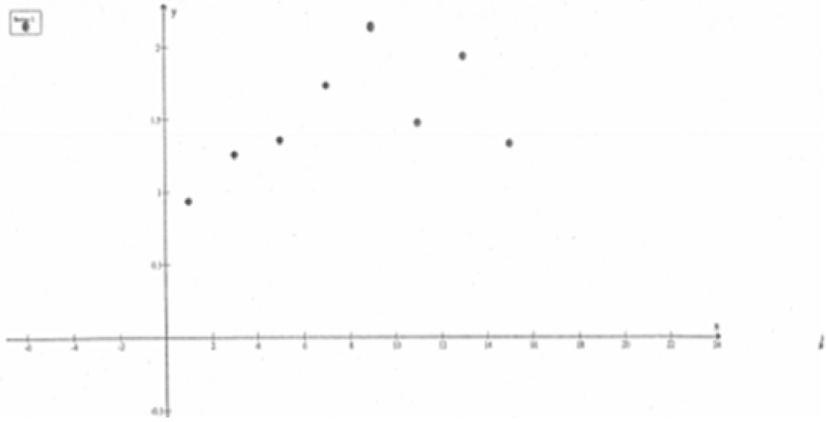
Mathematics

Work sample 1:
Maths assignment

It is clear on the graph below that on average with a wing length of 9cm the whirlybird stayed in flight for the longest period of time from the tests conducted. It is also clear that a whirlybird with a length of 1cm on average stayed in flight for the shortest period of time. *(Figure 3)*



Blow is a scatter plot graph of the data from the whirlybird experiment. Refer to *(Figure 4)*



Annotations

Interprets data to draw reasonable conclusions.

Constructs a scatter graph of data (using appropriate technology).

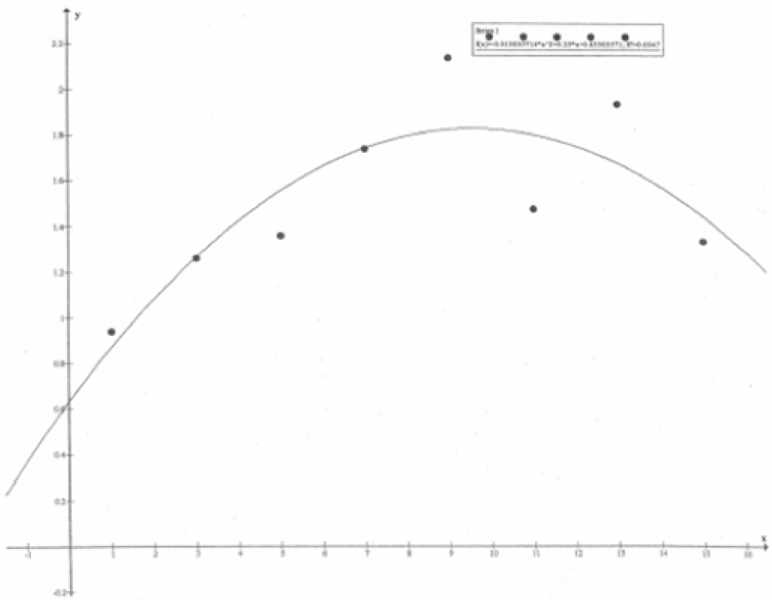
Mathematics

Work sample 1:
Maths assignment

Modelling and problem solving

Question 1

Below is a picture of a scatter plot of the averaged data results. *(Figure 5)*



The function of my graph in extended version is: $f(x)= -0.013035714x^2 + 0.25x + 0.63303571$: R2=0.6947

The shortened version of this function is: $x= -0.01x^2 + 0.25x + 0.6$

By using the short version of the quadratic function one can determine the turning point of the graph by following the method : x= -b/2a. In my function -b would be –(- and 2a would be .

Therefore $\frac{-0.25}{2^* -0.01} = 12.5$

Once one has found the turning point one can determine the length of the whirlybirds length that will stay in flight for the longest period of time. This length would be 12.5cm and by testing my

Annotations

Draws a curve of best fit for data, using appropriate technology.

Finds the equation of parabola, using appropriate technology.

Uses the equation to find the 'x' coordinate of the turning point and so answers the original question about length of whirly bird that gives maximum time in the air.

Mathematics

Work sample 1:
Maths assignment

predictions the whirlybird did in fact stay in the air for the longest period of time on an average of 2.21 seconds.

Part B

a) Can a different number of regions be formed with two tangents? How?

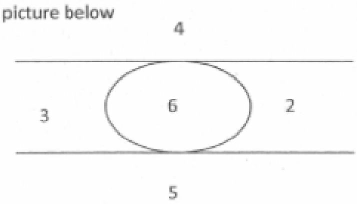
b) What is the least number of regions that can be formed with three tangents?

c) What is the greatest number of regions and the least number of regions that can be produced?

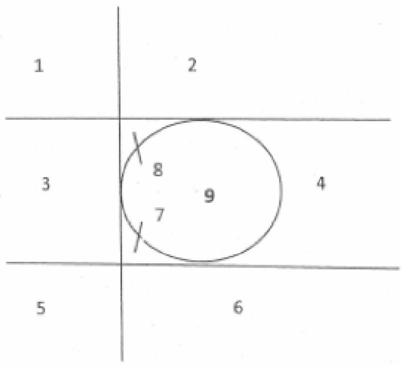
d) Using more tangents, investigate the greatest and least number of regions that can be produced. Display your results in a table.

Knowledge and Procedures

a) Yes a different number of regions can be formed with two tangents. This can be done by putting two parallel lines next to a circle so they do not join, since this would result in more regions. See picture below



b) The least number of tangents that can be formed with three tangents is 9. See picture below.



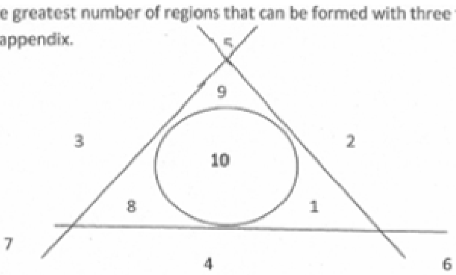
Annotations

Uses the concept of tangents and regions.

Mathematics

Work sample 1:
Maths assignment

c) The greatest number of regions that can be formed with three tangents is 10. See picture 3 in appendix.



d) Please refer to pictures 4-12 in appendix

(Figure 6)

Tangents	Maximum number of regions
1	3
2	6
3	10
4	15
5	21
6	28

(Figure 7)

Tangents	Minimum number of regions
1	3
2	5
3	9
4	13
5	19
6	25

Modelling & Problem Solving

By testing around with the different tangents and regions ranging up to six (tangents), one could discover that there was a pattern that the regions increased with by with every tangent. There was both a pattern for the maximum and minimum number of regions that could be found for each tangent. When there were 2 tangents the maximum number of regions was 6 and when there were 3 tangents the maximum number was 10. The first step to determining the number of tangents and reasons was to draw diagrams of the circles and tangents and write down the pattern. The next step was to place the maximum and minimum number of regions in a table. With x representing the number of tangents and y representing the maximum number of regions. Please refer to the table below *(Figure 8)*

X (tangents)	1	2	3	4	5	6
Y (max regions)	3	6	10	15	21	28

Annotations

Investigates systematically the relationship between the number of tangents and the number of regions.

Tabulates data.

Describes processes of investigation.

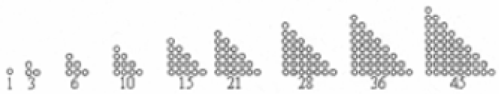
Mathematics

Work sample 1:
Maths assignment

One could then search the pattern on Google and determine that the pattern was in fact triangular numbers. This pattern is when the next number increases by one more than the last. For example 6+4=10 and then 10+5=15. The sequence of the triangular numbers comes from the natural numbers, if one always adds the next number, see example below

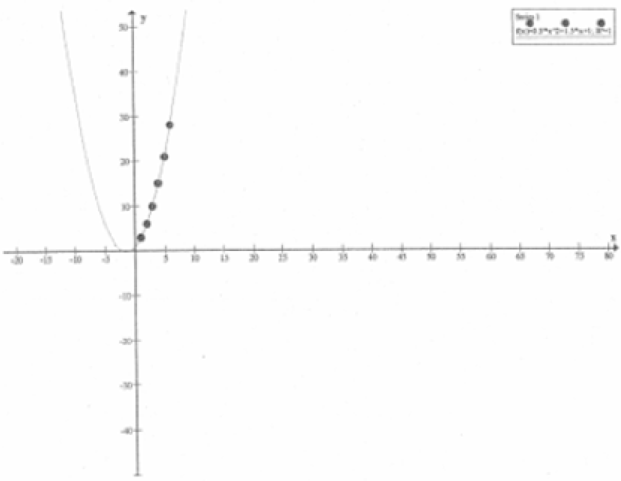
1
1+2=**3**
(1+2)+3=**6**
(1+2+3)+4=**10**
(1+2+3+4)+5=**15**
...

One can illustrate the name triangular number by the following drawing:



The information from the table above (Figure 8) was plotted onto a graph by using a graphing software. X (tangents) and y (maximum number of regions) were used as the variables. After plotting the tangents and regions (1-6) onto the graph the software automatically created a formula that related to the maximum number of regions and the number of tangents. The formula is: **$x = 0.5x^2 + 1.5x + 1$**. With x represents the number of tangents. I changed the formula to represent n (tangents) and thus the formula was created **$n = 0.5n^2 + 1.5n + 1$**. Refer to the graph below *(Figure 9)* to understand the formula and relationship better.

(Figure 9)



Annotations

Links to other contexts by referring to triangular numbers and Pascal's triangle.

Uses functions to predict later results.

Finds a quadratic expression to describe the relationship between the tangents and regions.

Connects data, algebraic functions and graphs.

Mathematics

Work sample 1:
Maths assignment

Using the relationship that was discovered above (**$n = 0.5n^2 + 1.5n + 1$**) one could now determine how many tangents would be needed to form 253 regions. By rearranging my formula one could find that:

Number of regions (R) = $0.5n^2+1.5n+1$ ——— When R=253: $253= 0.5n^2+1.5n+1$

Therefore: $0.5n^2+1.5n-252=0$ ——— (By using the quadratic formula)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

I found that a= 0.5 b= 1.5 c= -252

$$\text{therefore } x = \frac{-1.5 \pm \sqrt{1.5^2 - 4 \times 0.5 \times -252}}{2 \times 0.5}$$

Therefore $x= -1.5 \pm \sqrt{506.25}$ therefore $x= -1.5+ 22.5$ or $x=-1.5- 22.5$

So x= 21 or x = -24 Therefore **x=21** (ignore negative)

As a result 21 tangents formed a maximum of 253 regions. I checked my answer by subbing 21 into the question **$n = 0.5n^2 + 1.5n + 1$:** **$21=0.5(21)^2+ 1.5(21) + 1$** **21= 253**

The answer was double checked by going through the table below *(Figure 10)* and the answer is correct. Refer to table below.

(Figure 10)

Tangents	Maximum number of regions	Increased by
1	3	
2	6	3
3	10	4
4	15	5
5	21	6
6	28	7
7	36	8
8	45	9
9	55	10

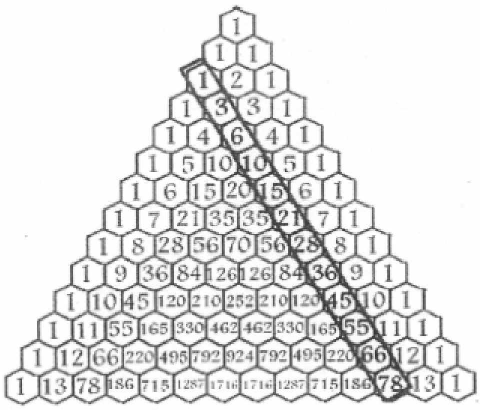
Annotations

Uses knowledge of quadratic functions to solve problems.

Mathematics

Work sample 1:
Maths assignment

10	66	11
11	78	12
12	91	13
13	105	14
14	120	15
15	136	16
16	153	17
17	171	18
18	190	19
19	210	20
20	231	21
21	253	22



(Figure 11)

The numbers in the table are all triangle numbers meaning that they increase by one more than the last.

To determine a relationship which gives the minimum number of regions for an even number of tangents one could place the values in a table and then plot the values on a graph by using a graphing software. See graph below *(Figure 11)*.

Annotations

Investigates and models an authentic situation and formulates a general formula.

Mathematics

Work sample 1:
Maths assignment

Appendix 1:

By checking the answers for the maximum number of regions for tangents ranging from 1 to 6 one must sub the tangent and maximum number of regions into the equation **$n = 0.5n^2 + 1.5n + 1$**. With n= representing the tangents and the number equalling the regions.

For 1 tangents and 3 maximum number of regions:

$n = 0.5n^2 + 1.5n + 1$ $1=0.5(1)^2+1.5(1)+1$ 1=3 so therefore answer is correct

For 2 tangents and 6 maximum number of regions:

$n = 0.5n^2 + 1.5n + 1$ $2=0.5(2)^2+1.5(2)+1$ 2=6 so therefore answer is correct

For 3 tangents and 10 maximum number of regions:

$n = 0.5n^2 + 1.5n + 1$ $3=0.5(3)^2+1.5(3)+1$ 3=10 so therefore answer is correct

For 4 tangents and 15 maximum number of regions:

$n = 0.5n^2 + 1.5n + 1$ $4=0.5(4)^2+1.5(4)+1$ 4=15 so therefore answer is correct

For 5 tangents and 21 maximum number of regions:

$n = 0.5n^2 + 1.5n + 1$ $5=0.5(5)^2+1.5(5)+1$ 5=21 so therefore answer is correct

For 6 tangents and 28 maximum number of regions:

$n = 0.5n^2 + 1.5n + 1$ $6=0.5(6)^2+1.5(6)+1$ 6=28 so therefore answer is correct

By checking the answers for the minimum number of regions for even numbers (2,4,6) one must sub the tangent and minimum number of regions into the equation **$x=0.5x^2+1x+1$**. With x= representing the tangents and the number equalling the regions.

For 2 tangents and 5 minimum number of regions:

$x=0.5x^2+1x+1$ $2=0.5(2)^2+1(2)+1$ 2=5 so therefore the answer is correct

For 4 tangents and 13 minimum number of regions:

$x=0.5x^2+1x+1$ $4=0.5(4)^2+1(4)+1$ 4=13 so therefore the answer is correct

For 6 tangents and 25 minimum number of regions:

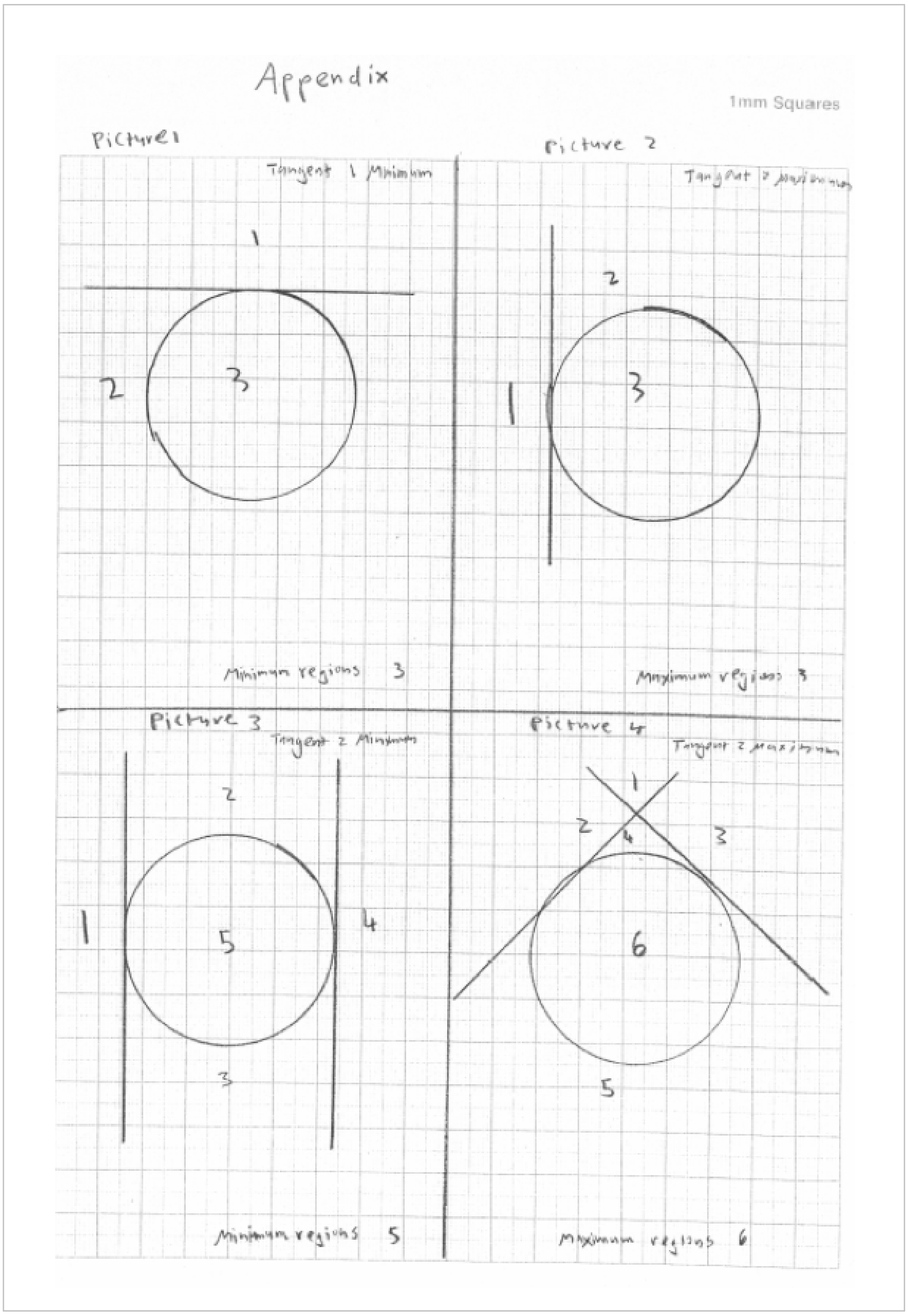
$x=0.5x^2+1x+1$ $6=0.5(6)^2+1(6)+1$ 6=25 so therefore the answer is correct

Annotations

Tests formula rigorously.

Mathematics

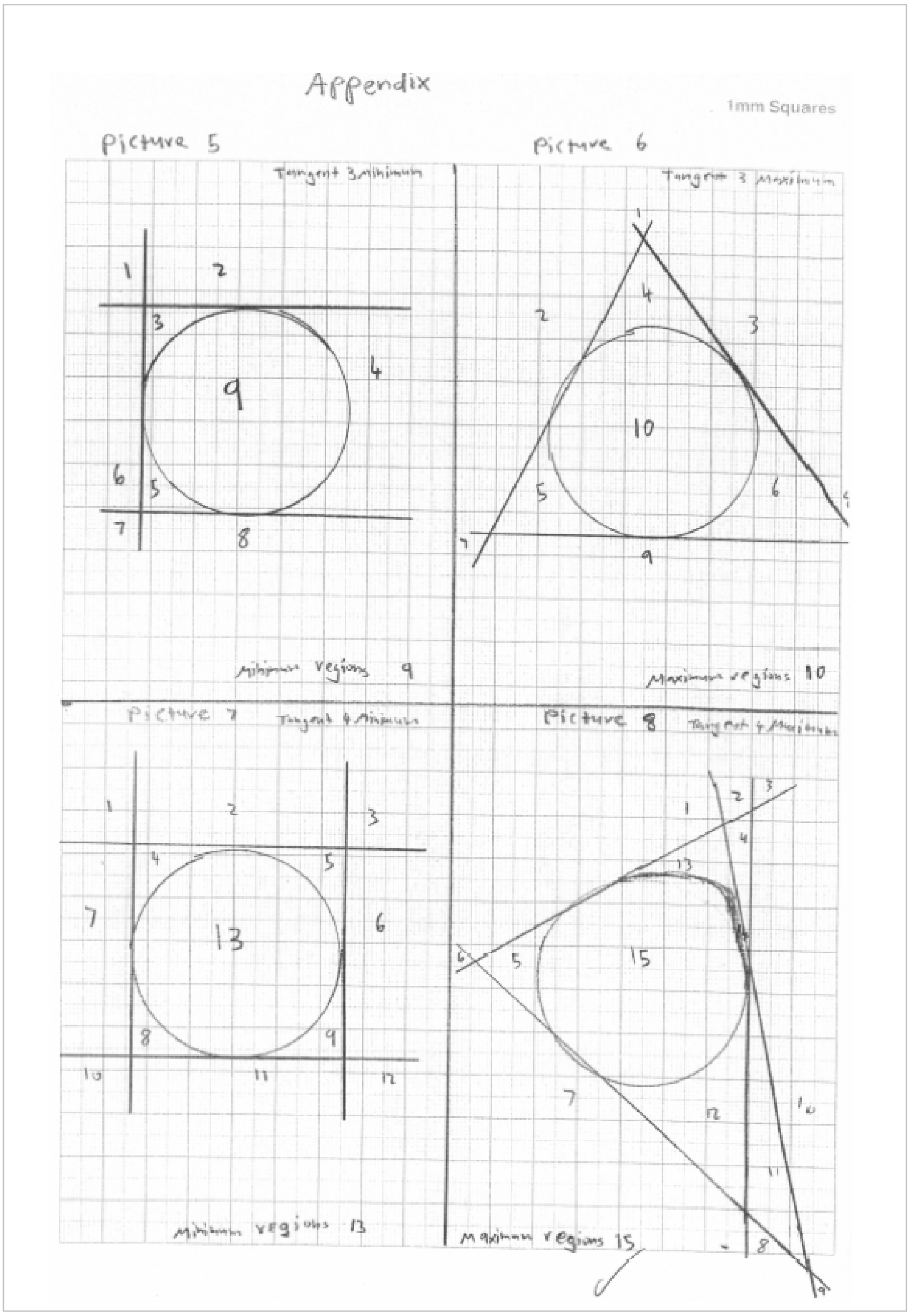
Work sample 1:
Maths assignment



Annotations

Provides evidence of investigative process.

Work sample 1:
Maths assignment

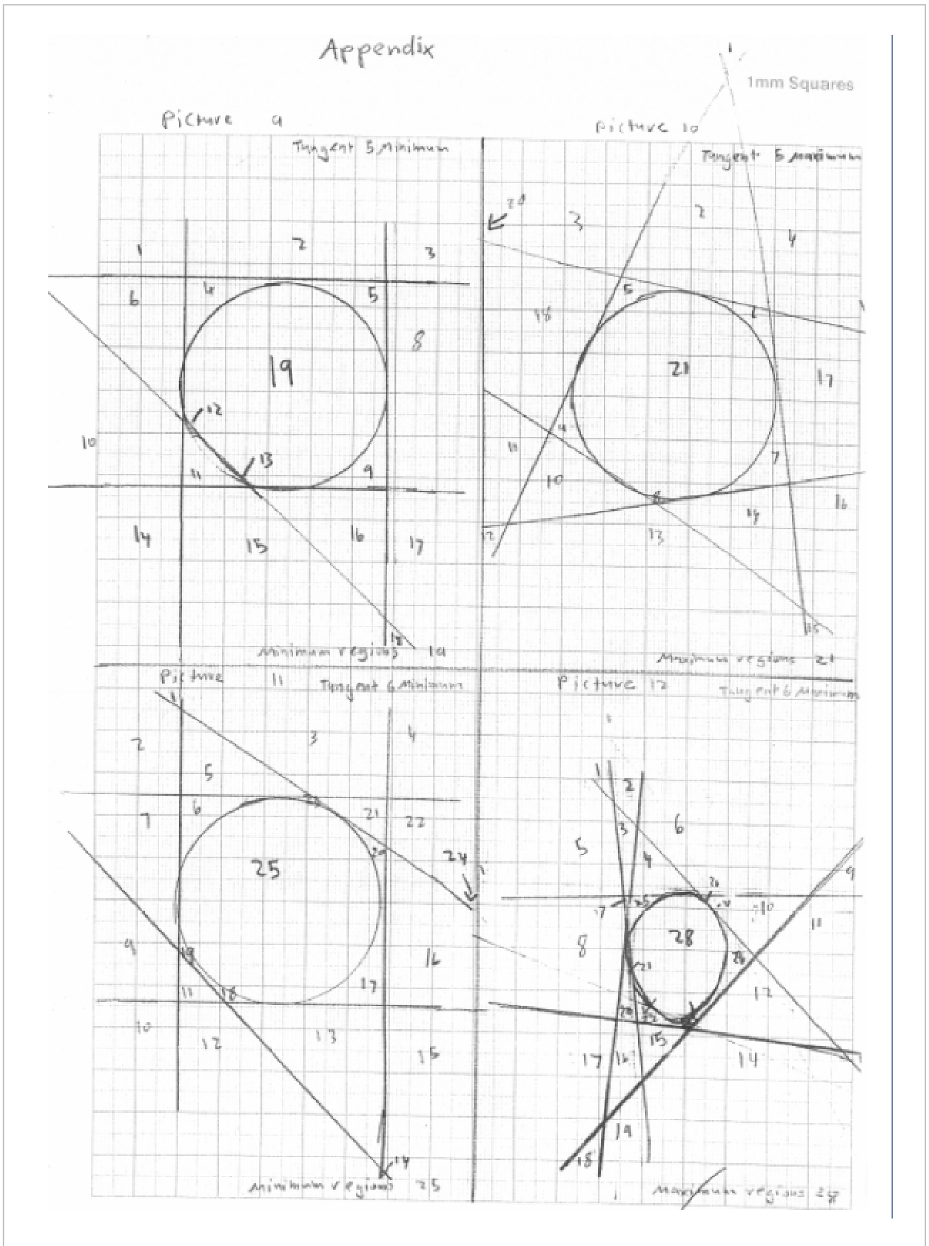


Annotations

Provides evidence of investigative process.

Mathematics

Work sample 1:
Maths assignment



Annotations

Provides evidence of investigative process.

Acknowledgment
ACARA acknowledges the contribution of trial school teachers and students for providing the tasks and work samples. The annotations are referenced to the Australian Curriculum achievement standards.

Mathematics

Work sample 2:
Number plane graphs

Relevant parts of the achievement standard

By the end of Year 10, students recognise the connection between simple and compound interest. They solve problems involving linear equations and inequalities. They make the connections between algebraic and graphical representations of relations. Students solve surface area and volume problems relating to composite solids. They recognise the relationships between parallel and perpendicular lines. Students apply deductive reasoning to proofs and numerical exercises involving plane shapes. They compare data sets by referring to the shapes of the various data displays. They describe bivariate data where the independent variable is time. Students describe statistical relationships between two continuous variables. They evaluate statistical reports.

Students expand binomial expressions and factorise monic quadratic expressions. They find unknown values after substitution into formulas. They perform the four operations with simple algebraic fractions. Students solve simple quadratic equations and pairs of simultaneous equations. They use triangle and angle properties to prove congruence and similarity. Students use trigonometry to calculate unknown angles in right-angled triangles. Students list outcomes for multi-step chance experiments and assign probabilities for these experiments. They calculate quartiles and inter-quartile ranges.

Summary of task

Students have been learning about non-linear relationships. In this task with guided questions they were asked to:

- complete tables and values
- draw graphs of non-linear relationships using their tables of values
- explain features of graphs and equations
- match graphs to their equations
- use the correct names for non-linear relations.

Mathematics

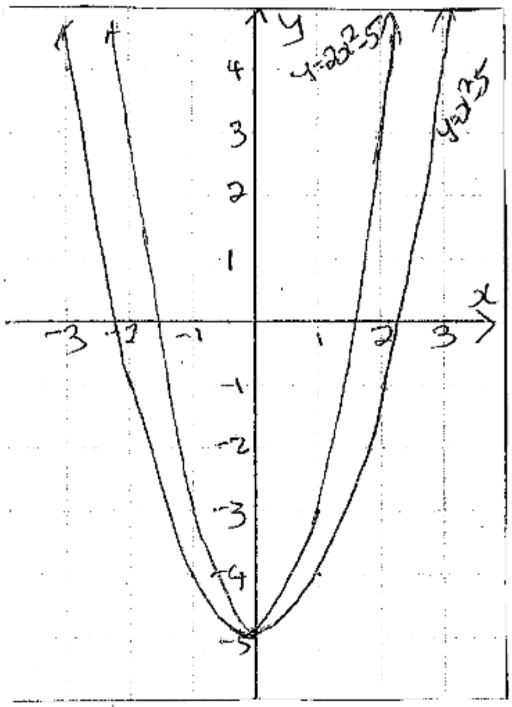
Work sample 2:
Number plane graphs

1.

(a) Complete the table of values for the graph $y = x^2 - 5$.

x	-3	-2	-1	0	1	2	3
y							

(b) Plot the points from the table above to graph $y = x^2 - 5$.



(c) Explain using words and/or the number plane above how the graph with equation $y = x^2 - 5$ would differ from the graph of $y = 2x^2 - 5$.

x	-3	-2	-1	0	1	2	3
y	13	3	-3	-5	-3	3	13

The 2 before the x^2 makes the graph steeper.

Annotations

Completes tables of values for parabolas.

Plots points to graph parabolas.

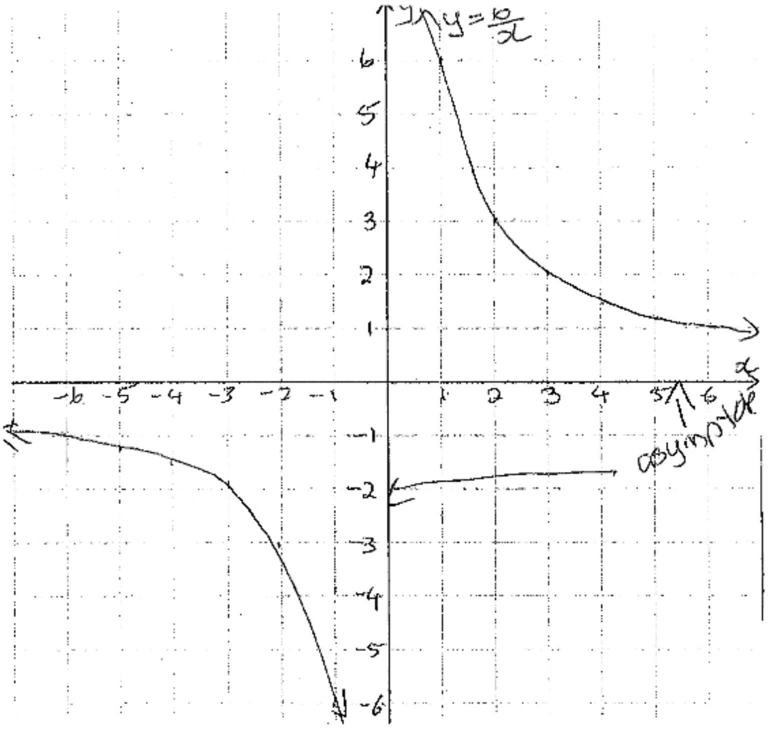
Explains the effect on the graph of the coefficient 2 before x^2 after completing a table of values for $y = 2x^2 - 5$ and graphing the parabola.

Mathematics

Work sample 2:
Number plane graphs

2 (a) Complete the following table of values and use it to graph $y = \frac{6}{x}$ on the set of axes below.

x	-6	-4	-3	-2	-1	0	1	2	3	4	6
y	-1	-1.5	-2	-3	-6	----	6	3	2	1.5	1



(a) What is the name given to this type of graph? hypebola.

(b) Explain what an asymptote is.
asymptote when the line can't touch the x or y axes.

(c) Indicate an asymptote on the graph above.

(d) What is the equation of the asymptote you indicated above?
$x = 0$, $y = 0$

Annotations

Completes tables for values for $y = \frac{6}{x}$

Names the graph as a hyperbola.

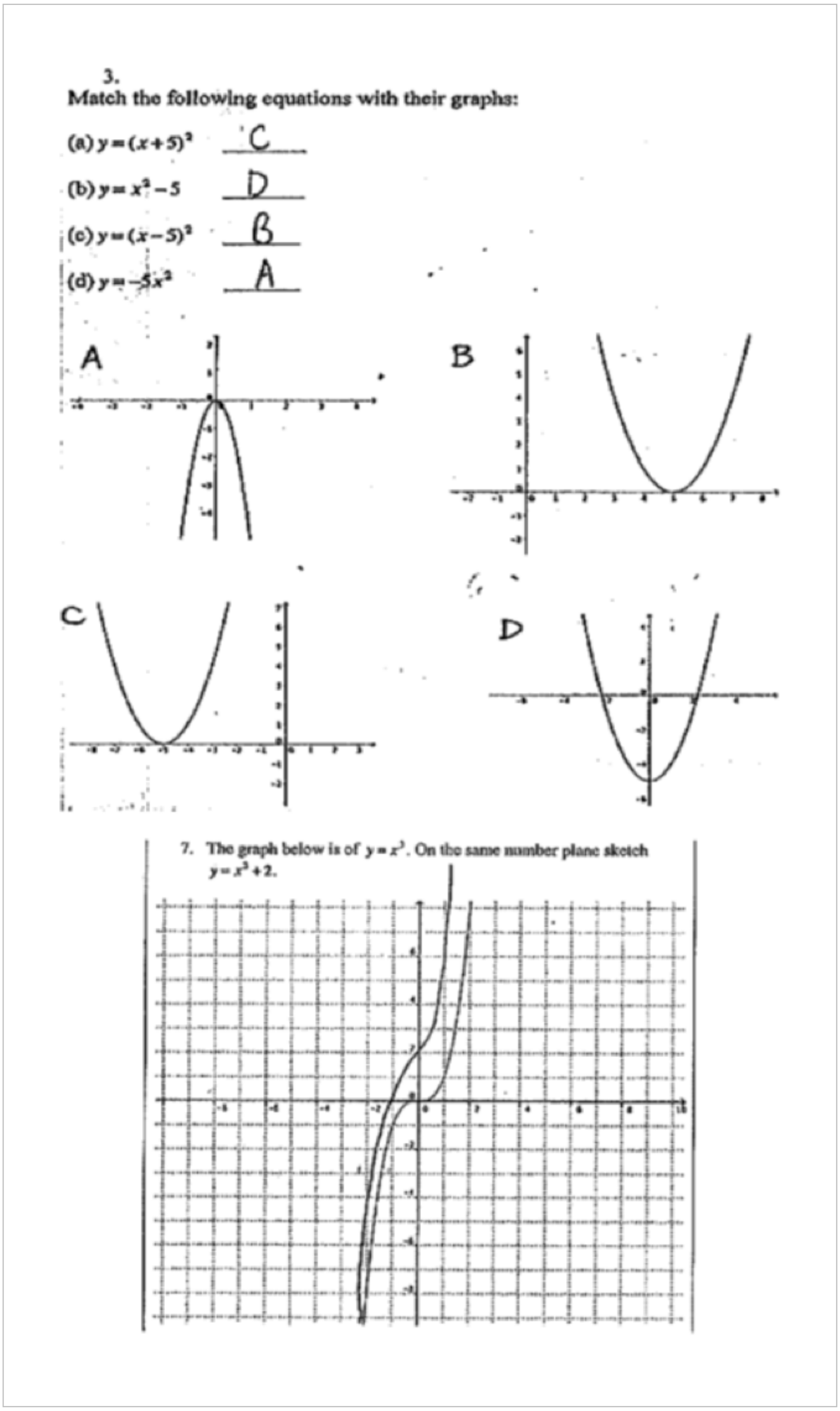
Explains what the asymptote is for this hyperbola.

Shows asymptotes on this hyperbola.

Gives equations for asymptotes for this hyperbola.

Mathematics

Work sample 2:
Number plane graphs



3.
Match the following equations with their graphs:

(a) $y=(x+5)^2$ C

(b) $y=x^2-5$ D

(c) $y=(x-5)^2$ B

(d) $y=-5x^2$ A

7. The graph below is of $y=x^3$. On the same number plane sketch $y=x^3+2$.

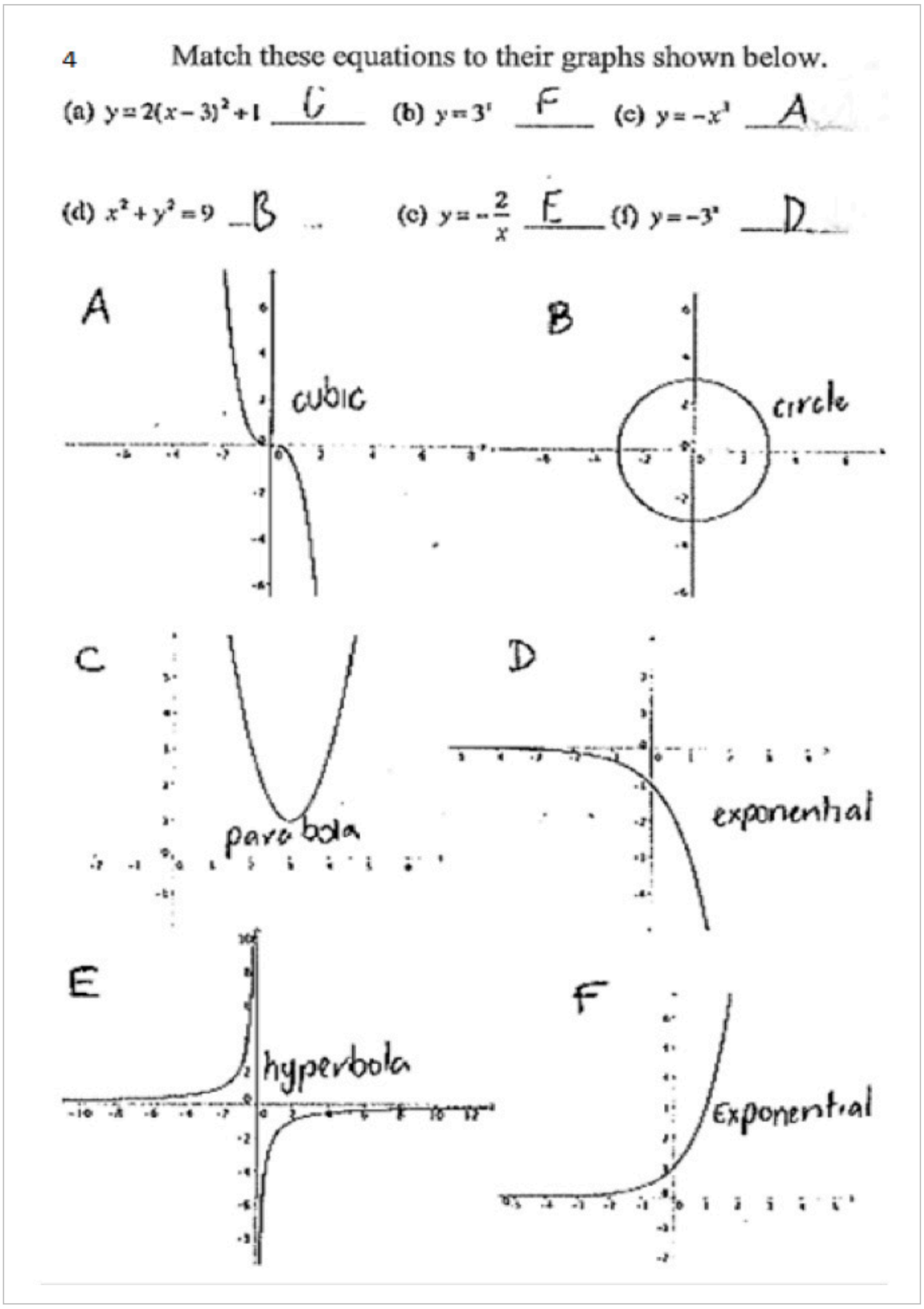
Annotations:

Matches graphs of parabolas to their equations.

Translates the graph $y=x^3$ *up 2 units to give the graph of* $y=x^3+2$

Mathematics

Work sample 2:
Number plane graphs


4 Match these equations to their graphs shown below.

(a) $y = 2(x-3)^2 + 1$ C (b) $y = 3^x$ F (c) $y = -x^3$ A

(d) $x^2 + y^2 = 9$ B (e) $y = -\frac{2}{x}$ E (f) $y = -3^x$ D

Annotations

Matches graphs of a variety of relations to their equations.

Matches graphs to their correct name.

Acknowledgment
ACARA acknowledges the contribution of trial school teachers and students for providing the tasks and work samples. The annotations are referenced to the Australian Curriculum achievement standards.

Mathematics

Work sample 3:
Linear equations – Taxi fares

Relevant parts of the achievement standard

By the end of Year 10, students recognise the connection between simple and compound interest. They solve problems involving linear equations and inequalities. They make the connections between algebraic and graphical representations of relations. Students solve surface area and volume problems relating to composite solids. They recognise the relationships between parallel and perpendicular lines. Students apply deductive reasoning to proofs and numerical exercises involving plane shapes. They compare data sets by referring to the shapes of the various data displays. They describe bivariate data where the independent variable is time. Students describe statistical relationships between two continuous variables. They evaluate statistical reports.

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Summary of task

Students were required to use their knowledge of linear functions to answer questions about the fares charged by two taxi companies. They were asked to:

- represent the relationship between cost and distance travelled in different forms, including algebraic equations
- use appropriate strategies to solve problems based on taxi fares.

Mathematics

Work sample 3:
Linear equations – Taxi fares

Complete all the questions on your own paper and show all your working.

ANNIE'S TAXI

Useful definitions

- Flag fall — a fixed cost to hire the taxi.
- Distance charge — a charge for each kilometre travelled.

Annie's Taxi uses the following schedule for calculating fares:

	Amount
Flag fall	$2.50
Distance charge ($/km)	$1.70 per km

1. a. Copy the table below onto your own paper.

   b. Complete the row labelled **Cost ($)** by calculating the cost of trips for different distances, using the information in the table above.

Distance (km)	1	5	12	50
Cost ($)				

2. Write a word equation for how you calculated the cost in Question 1b.
3. Write an algebraic equation for the cost of travel with Annie's Taxi. State what the symbols you use represent.
4. Graph the data from the table you completed in Question 1b. Place distance on the x-axis.
5. a. Use your graph to estimate how far you can travel for $82.00.

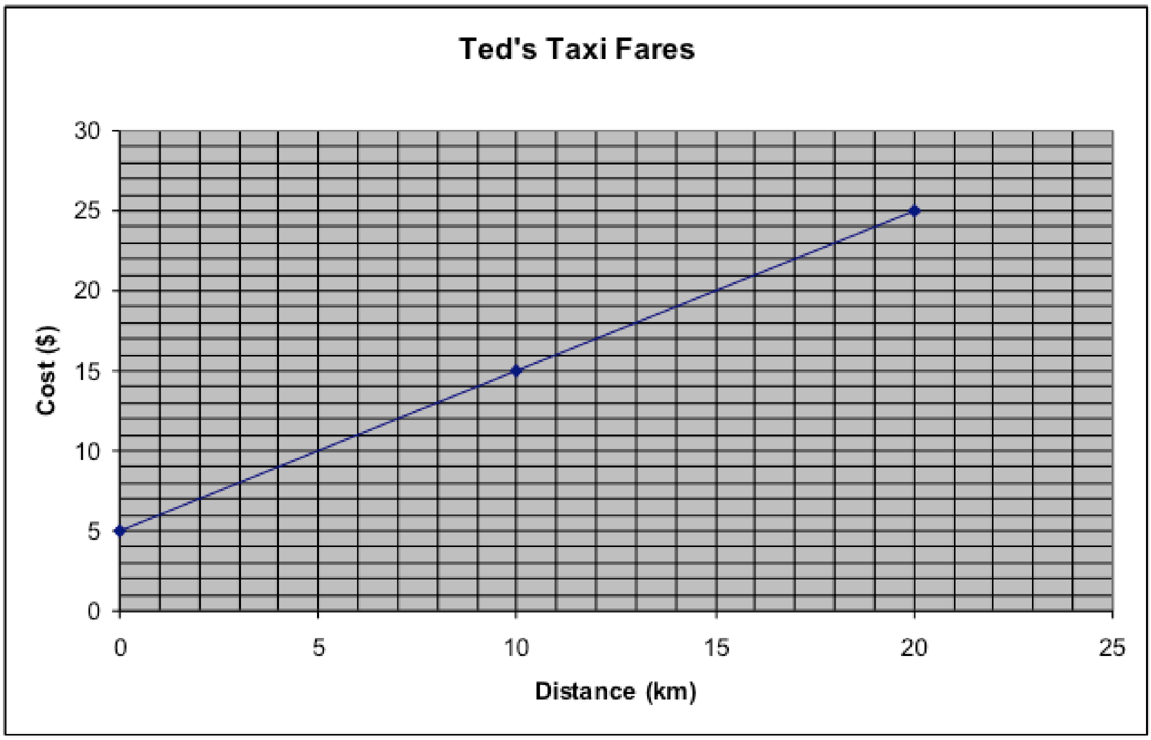
   b. Use a different method to check the accuracy of your estimate.

Mathematics

Work sample 3:
Linear equations – Taxi fares

TED'S TAXI

Ted's Taxi offers a fare schedule different from that of Annie's Taxi. The graph below shows the cost for trips of different distances.



6. a. Write an equation for this function.

   b. Determine the flag fall and distance charge ($/km) for Ted's Taxi.

7. Annie's Taxi and Ted's Taxi are the only two taxi companies in town. Ted's Taxi uses the advertising slogan "Travel with Ted — the best taxi fare in town".
   Is this claim true? Justify your answer mathematically.

Mathematics

Work sample 3:
Linear equations – Taxi fares

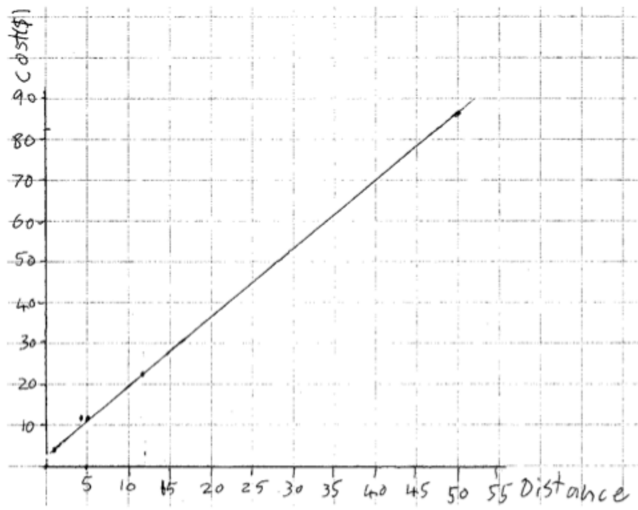
①a.

Distance (Km)	1	5	12	50
Cost ($)	4.2	11	22.9	87.5

② (Cost $) = 1.7 x Distance + flag fall

③ C = 1.7 . d + 5.0

④ see graph



⑤ a. 47km : estimate

B.) You could travel 46km with $1.30 leftover = (1.70 x 46) + 2.50.

6. a) y = m.x + c

c = (m) x d + c

c = 1 . d + 5

b) Flag Fall = 5

Distance = 1

Charge

Annotations

Completes table of values using given information.

Finds the equation of a linear relationship.

Graphs a linear relationship from a table of values.

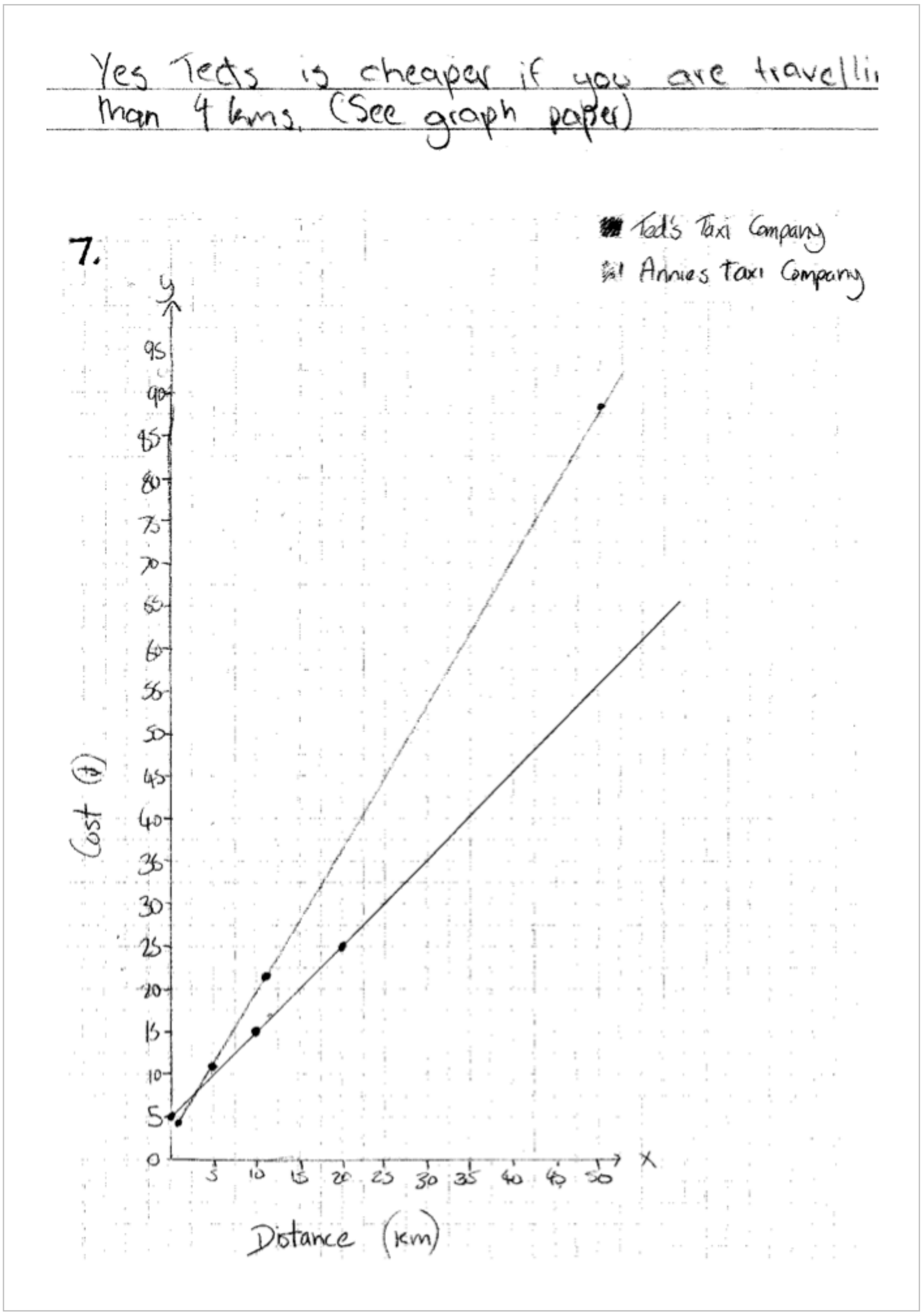
Finds independent variable from given dependent variable, using a graph.

Checks accuracy of an answer read from a graph using the equation of the linear relationship.

Identifies 'y' intercept as the constant term (flag fall) and gradient as the rate charged per km.

Mathematics

Work sample 3:
Linear equations – Taxi fares



Annotations

Models an authentic situation to solve a problem.

Supports conclusions using appropriate mathematical reasoning.

Acknowledgment
ACARA acknowledges the contribution of trial school teachers and students for providing the tasks and work samples. The annotations are referenced to the Australian Curriculum achievement standards.

Mathematics

Work sample 4:
Similar or congruent triangles

Relevant parts of the achievement standard

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Summary of task

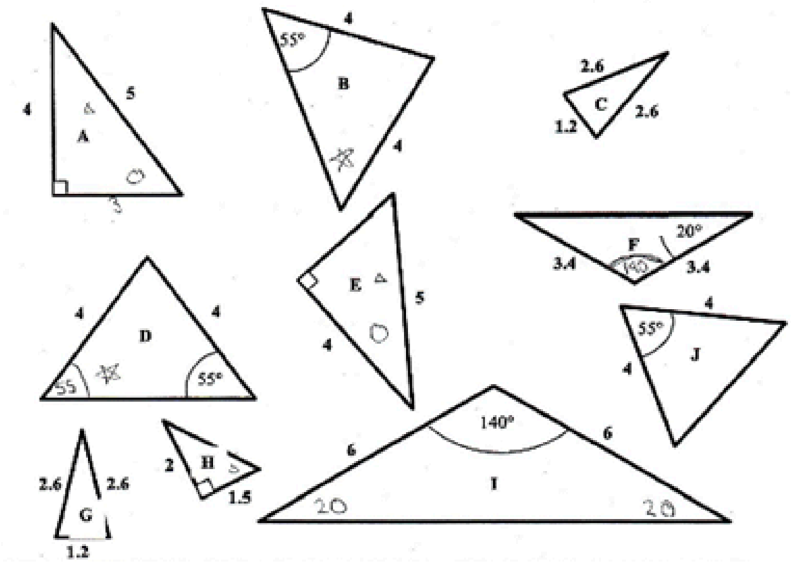
Students were given a worksheet showing a variety of triangles, and they indicated which triangles were similar and which triangles were congruent, giving reasons. Students then answered several questions to demonstrate their understanding of similarity and congruence.

Mathematics

Work sample 4:
Similar or congruent triangles

Similar? Congruent?

1. Consider the following triangles and complete the table below. (All lengths are in centimetres)



Which triangles are similar?	Which triangles are congruent?	Reasons for congruency
A, H	D & B	base angles of isol Δ equal. AAS
F, I	A, E	RHS
H, E	C, G	SSS
	J	

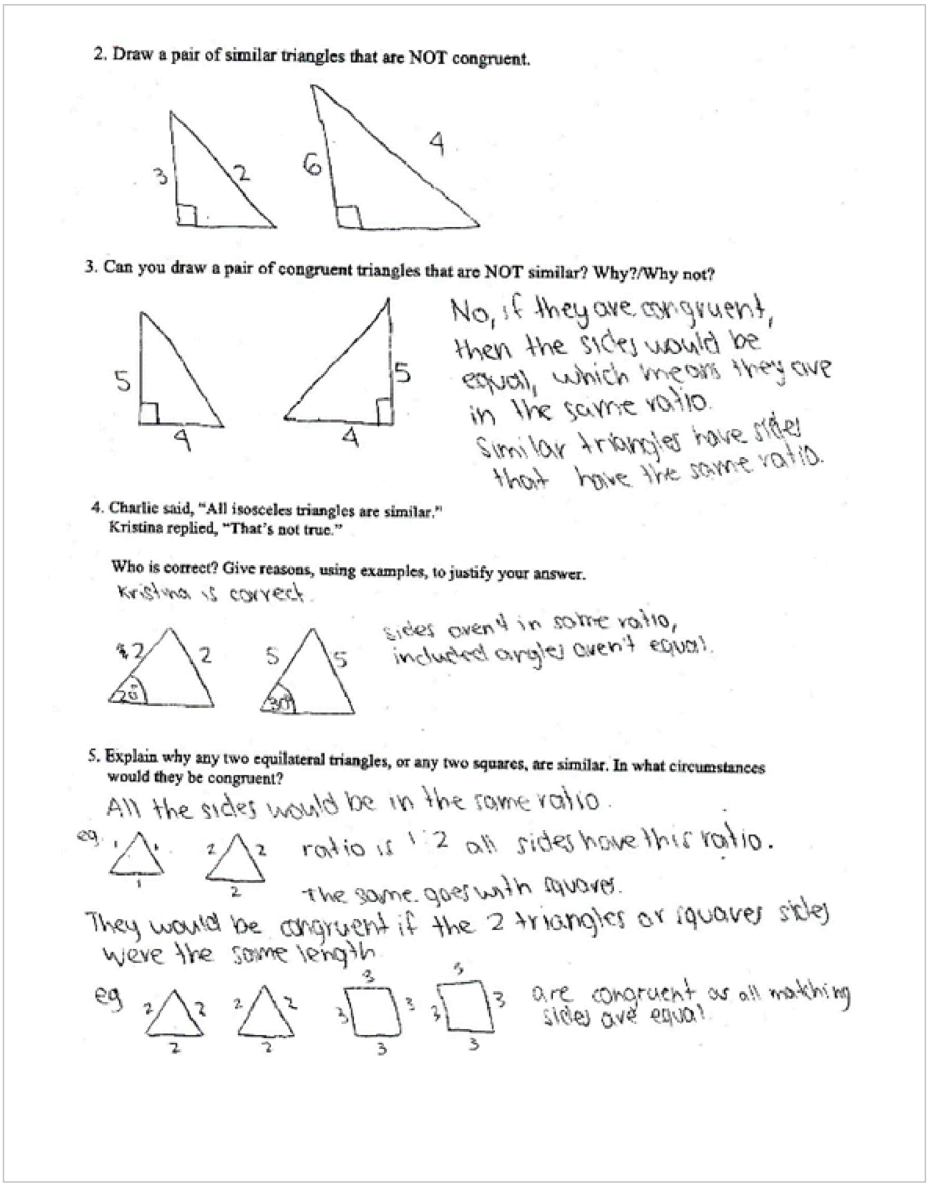
Annotations

Identifies pairs of congruent triangles.

Identifies pairs of similar triangles.

Mathematics

Work sample 4:
Similar or congruent triangles


2. Draw a pair of similar triangles that are NOT congruent.

3. Can you draw a pair of congruent triangles that are NOT similar? Why?/Why not?

4. Charlie said, "All isosceles triangles are similar."
Kristina replied, "That's not true."

Who is correct? Give reasons, using examples, to justify your answer.

5. Explain why any two equilateral triangles, or any two squares, are similar. In what circumstances would they be congruent?

Annotations

Demonstrates a clear understanding of congruency and similarity.

Uses appropriate mathematical reasoning, including informal deductive reasoning, to support statements regarding isosceles triangles, regular polygons and similarity.

Acknowledgment
ACARA acknowledges the contribution of the NSW Board of Studies for providing the tasks and work samples. The annotations are referenced to the Australian Curriculum achievement standards.

Mathematics

Work sample 5:
Trigonometry assignment

Relevant parts of the achievement standard

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Summary of task

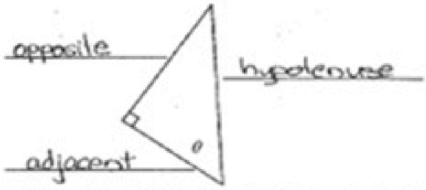
Students demonstrated their understanding of basic right-angled triangle trigonometry by labelling parts sides of a trigonometric and writing out the ratios for a given triangle. They applied their knowledge to problems involving angles of elevation and depression and three figure bearings.

Mathematics

Work sample 5:
Trigonometry assignment

TRIGONOMETRY ASSIGNMENT

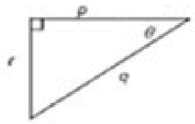
1a Label the sides of this triangle correctly in relation to the angle *θ*, using the words, *hypotenuse, adjacent and opposite.*



1b. Complete the following statements about trigonometric ratios in right angled triangles:

sin = opposite / hypotenuse cos = adjacent / hypotenuse tan = opposite / adjacent

2. Write out the correct ratio statements for this triangle below.



sin *θ* = r / q

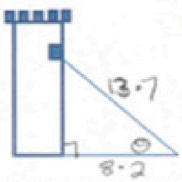
cos *θ* = p / q

tan *θ* = r / p

Princess Ottline has been locked in a vertical tower by her evil stepmother. The Charming Prince Percival comes to her rescue. He leans his trusty ladder against the tower so it just reaches the tower's only window. The ladder is 13.7 metres long and the foot of the ladder is 8.2 metres from the base of the tower. You may assume that the tower is perpendicular to the ground.

a. Complete the diagram below, marking in all known measurements and the angle between the ladder and the ground.

θ



b. Find the angle the ladder makes with the ground, correct to the nearest degree.

c = a / h   cos θ = 8·2 / 13·7

= 0·5985...

θ = 53·2345...

The ladder makes an angle of approximately 53° with the ground

Annotations

Labels sides of a right-angled triangle in relation to a marked angle.

States trigonometric ratios in terms of sides in right-angled triangles.

Finds the sin, cos and tan ratios for a specific right-angled triangle.

Completes diagram from a written description.

Marks side lengths.

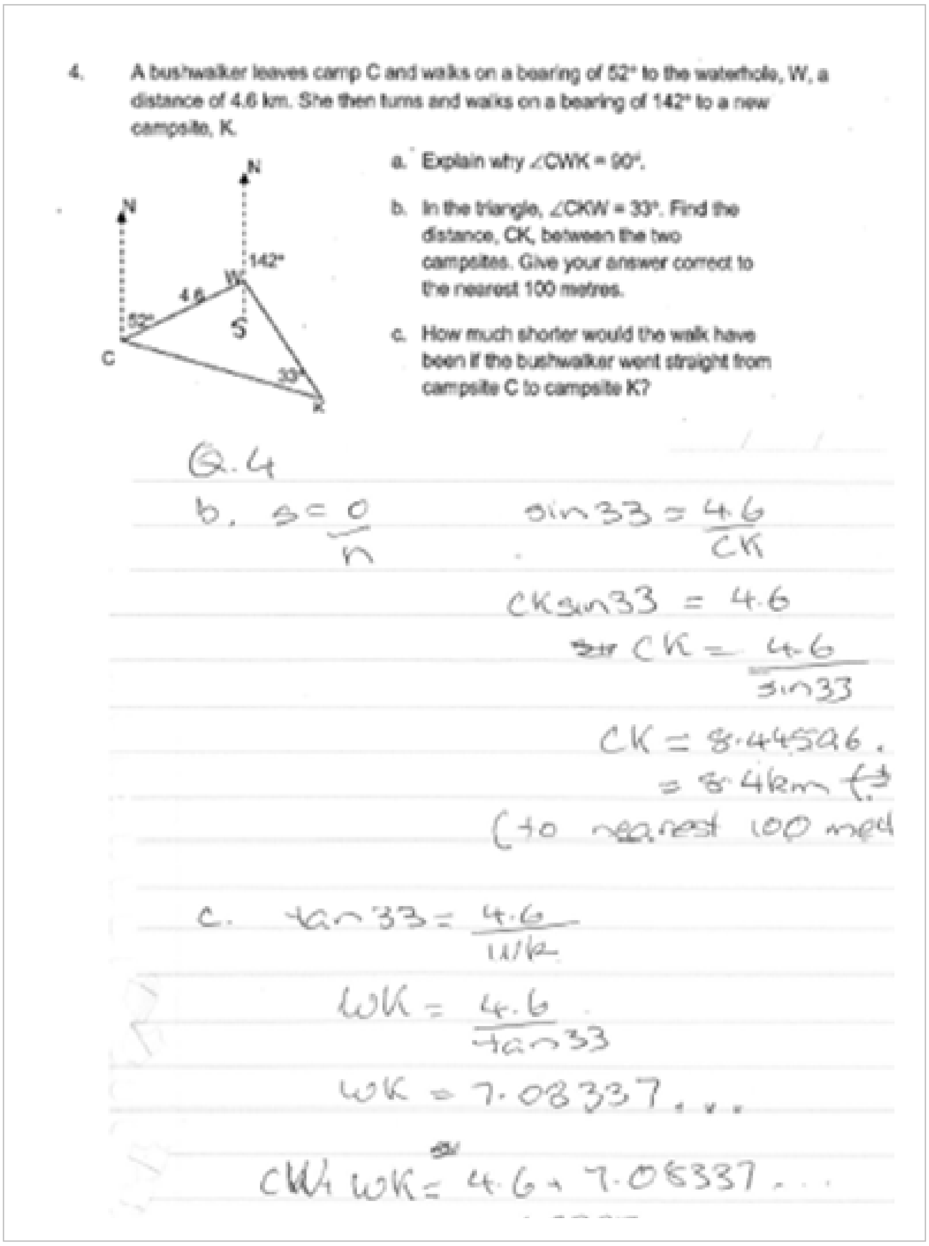
Chooses correct trigonometric ratios.

Finds a required angle.

Approximates an angle correct to the nearest degree.

Mathematics

Work sample 5:
Trigonometry assignment


4. A bushwalker leaves camp C and walks on a bearing of 52° to the waterhole, W, a distance of 4.6 km. She then turns and walks on a bearing of 142° to a new campsite, K.

a. Explain why ∠CWK = 90°.

b. In the triangle, ∠CKW = 33°. Find the distance, CK, between the two campsites. Give your answer correct to the nearest 100 metres.

c. How much shorter would the walk have been if the bushwalker went straight from campsite C to campsite K?

Annotations

Uses geometric reasoning to explain a given fact.

Identifies correct trigonometric ratio.

Finds a required side length.

Recognises the need for subtraction to answer a question without recognising the need to calculate another side length to find total distance travelled.

Acknowledgment
ACARA acknowledges the contribution of trial school teachers and students for providing the tasks and work samples. The annotations are referenced to the Australian Curriculum achievement standards.

Mathematics

Work sample 6:
Algebraic expressions

Relevant parts of the achievement standard

By the end of Year 10, students recognise the connection between simple and compound interest. They solve problems involving linear equations and inequalities. They make the connections between algebraic and graphical representations of relations. Students solve surface area and volume problems relating to composite solids. They recognise the relationships between parallel and perpendicular lines. Students apply deductive reasoning to proofs and numerical exercises involving plane shapes. They compare data sets by referring to the shapes of the various data displays. They describe bivariate data where the independent variable is time. Students describe statistical relationships between two continuous variables. They evaluate statistical reports.

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Summary of task

Students completed a worksheet on expanding algebraic expressions using the four operations.

Mathematics

Work sample 6:
Algebraic expressions

ALGEBRA

Expand and simplify these expressions. Look at the example first which shows you how to set them out. Show working.

EXAMPLE

$5(x+3)-2(x-7)-12$
$=5x+15-2x+7-12$
$=3x+29-12$
$=3x+17$

Questions

1. $5(x+3)+2(x-7) = 5x+15+2x-14 = 7x+1$
2. $3(x-5)+4(x+6) = 3x-15+4x+24 = 7x+9$
3. $3(2x-1)+2(3x+4)$ $6x-3+6x+8 = 12x+5$
4. $4(5x+2)+8(2x-1)$ $20x+8+16x-8 = 36x$
5. $7(x+1)-5(x+3)$ $7x+7-5x-15 = 2x-8$
6. $3(x+5)-2(x-7)$ $3x+15-2x+14 = x+29$
7. $4(3x-2)-5(2x-1)$ $12x-8-10x+5 = 2x-3$
8. $7(2x+1)-2(5x+4)$ $14x+7-10x-8 = 4x-1$
9. $3(5x-7)-7(2x-3)$ $15x-21-14x+21 = x$.
10. $6(3x+2)-9(2x-1)$ $18x+12-18x+9 = 21$
11. $5(x+4)+(x-8)$ $5x+20+x-8 = 6x+14$
12. $8(x-1)+(3x-4)$ $8x-8+3x-4 = 12x-12$
13. $3(2x-5)+(4x+9)$ $6x-15+4x+9 = 10x-6$
14. $2(x-5)-(x+3)$ $2x-10-x+3 = x-7$
15. $8(x-1)-(x-8)$ $8x-8-x+8 = 7x$
16. $5(3x+4)-(9x-4)$ $15x+20-9x+4 = 6x+24$
17. $7(2x-1)-(3x-7)$ $14x-7-3x+7 = 11x$
18. $4(2x-3)-(8x+5)$ $8x-12-8x-5 = -17$
19. $7+4(x-1)$ $7+4x-4 = 4x+3$
20. $5x-2(3x-1)$ $5x-6x+2 = -x+2$

The next 5 questions should be completed in exactly the same way but they are longer so set your work out carefully.

21. $8(x+2)+7(x-3)-5(x-4)$ $8x+16+7x-21-5x+4 = 10x+15$
22. $9+6(x-5)+(3x-8)$ $9+6x-30+3x-8 = 9x-29$
23. $2(x+3)-7(x-1)-(4x+15)$ $2x+6-7x+7-4x-15 = -9x-2$
24. $7x-(3x+2)+(2x-7)+10$ $7x-(6x+2)+2x-7)-10 = 6x+1$
25. $6(2x+5)-4(3x-4)-(2x+7)+2x$
$= 12x+30-12x+16-2x-14$
$+2x = 32$

Annotations

Correctly expands and simplifies algebraic expressions.

Acknowledgment

ACARA acknowledges the contribution of trial school teachers and students for providing the tasks and work samples. The annotations are referenced to the Australian Curriculum achievement standards.

Mathematics

Work sample 7:
Algebraic fractions

Relevant parts of the achievement standard

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Summary of task

Prior to this task, students had practice at manipulating algebraic fractions.

Students were required to simplify the algebraic fractions worksheet in a lesson.

Mathematics

Work sample 7:
Algebraic fractions

Complete the following calculations:

1. Simplify the following:

(a) $\frac{x}{3} + \frac{x}{4}$

(b) $\frac{2}{2xy} + \frac{4}{xy^3}$

(c) $\frac{3x+1}{2} - (6x+5)$

(d) $\frac{3}{b-1} - \frac{4}{b-2}$

(e) $\frac{\frac{2x+2}{y}}{\frac{x+1}{xy}}$

(f) $\frac{2}{x^2-4x} + \frac{4}{x-4}$

(g) $\frac{\frac{1}{x+1}}{2-1}$

(h) $\frac{1}{x+1} - \frac{1}{x+2} + \frac{1}{x+3}$

(i) $\frac{4a}{7} + \frac{3a+5}{2} - \frac{3(a+2)}{4}$

(j) $\frac{3p}{12} - (\frac{p}{2} - \frac{p}{4} + \frac{5p}{6})$

2. Simplify the follwing:

(a) $\frac{4(x+1)}{3} - \frac{5(x-2)}{2}$

(b) $\frac{x^2+3x}{x+4} \times \frac{2x+8}{5x}$

(c) $\frac{8x-24}{4} \div \frac{x+7}{12}$

(d) $\frac{y^2-6y}{y+5} \times \frac{3y+15}{2y-12}$

(e) $\frac{5m-7}{4m+8} \div \frac{m+2}{3m+6}$

(f) $\frac{6p-3}{4} \div \frac{-4p+2}{12}$

Mathematics

Work sample 7:
Algebraic fractions

Algebraic Fractions

1\. a) $\frac{x}{3} + \frac{x}{4}$

$= \frac{4x + 3x}{12}$

$= \frac{7x}{12}$

b) $\frac{2}{xy} + \frac{4}{xy^3} = \frac{2y^2 + 4}{xy^3}$

$= \frac{2(y^2+2)}{xy^3}$

c) $\frac{3x+1}{2} - (6x+5)$

$= \frac{3x+1 - 2(6x+5)}{2}$

$= \frac{3x+1 - 12x - 10}{2}$

$= \frac{-9x - 9}{2}$

$= \frac{-9(x+1)}{2}$

d) $\frac{3}{b-1} - \frac{4}{b-2}$

$= \frac{3(b-2) - 4(b-1)}{(b-1)(b-2)}$

$= \frac{3b - 6 - 4b + 4}{(b-1)(b-2)}$

$= \frac{-b-2}{(b-1)(b-2)}$

e) $\dfrac{\frac{2x+2}{y}}{\frac{x+1}{xy}}$

$= \frac{2x+2}{y} \div \frac{x+1}{xy}$

$= \frac{2x+2}{y} \times \frac{xy}{x+1}$

$= \frac{2(x+1)}{y} \times \frac{xy}{(x+1)}$

$= 2x$

f) $\frac{2}{x^2-4x} + \frac{4}{x-4}$

$= \frac{2}{x(x-4)} + \frac{4}{(x-4)}$

$= \frac{2 + 4x}{x(x-4)}$

$= \frac{2(1 + 2x)}{x(x-4)}$

g) $\dfrac{\frac{1}{x+1}}{2-1} = \dfrac{\frac{1}{x+1}}{1}$

$= \frac{1}{x+1}$

Annotations

Correctly simplifies the algebraic fractions with numerical denominators.

Factorises algebraic expressions by taking out a negative common factor.

Correctly factorises algebraic expressions to enable cancelling down of algebraic fraction.

Mathematics

Work sample 7:
Algebraic fractions

h) $\frac{1}{x+1} - \frac{1}{x+2} + \frac{1}{x+3}$

$= \frac{(x+2)(x+3) - (x+1)(x+3) + (x+1)(x+2)}{(x+1)(x+2)(x+3)}$

$= \frac{(x+3)[(x+2) - (x+1)] + (x+1)(x+2)}{(x+1)(x+2)(x+3)}$

$= \frac{(x+3)(x+2-x-1) + (x+1)(x+2)}{(x+1)(x+2)(x+3)}$

$= \frac{(x+3) + (x^2+3x+2)}{(x+1)(x+2)(x+3)}$

i) $\frac{4a}{7} + \frac{3a+5}{2} - \frac{3(a+2)}{4}$

$= \frac{4(4a) + 14(3a+5) - 21(a+2)}{28}$

$= \frac{16a + 42a + 70 - 21a - 42}{28}$

$= \frac{37a + 28}{28}$

j) $\frac{3p}{12} - \left(\frac{p}{2} - \frac{p}{4} + \frac{5p}{6}\right)$

$= \frac{3p - 6p + 3p - 10p}{12}$

$= \frac{-10p}{12}$

$= \frac{-5p}{6}$

2. a) $\frac{4(x+1)}{3} - \frac{5(x-2)}{2}$

$= \frac{8(x+1) - 15(x-2)}{6}$

$= \frac{8x+8 - 15x + 30}{6}$

$= \frac{-7x + 38}{6}$

b) $\frac{x^2+3x}{x+4} \times \frac{2x+8}{5x}$

$= \frac{x(x+3)}{(x+4)} \times \frac{2(x+4)}{5x}$

$= \frac{2(x+3)}{5}$

c) $\frac{8x-24}{4} \div \frac{x+7}{12}$

$= \frac{8(x-3)}{4} \times \frac{12}{(x+7)}$

$= \frac{24(x-3)}{(x+7)}$

d) $\frac{y^2-6y}{y+5} \times \frac{3y+15}{2y-12}$

$= \frac{y(y-6)}{(y+5)} \times \frac{3(y+5)}{2(y-6)}$

$= \frac{3y}{2}$

Annotations

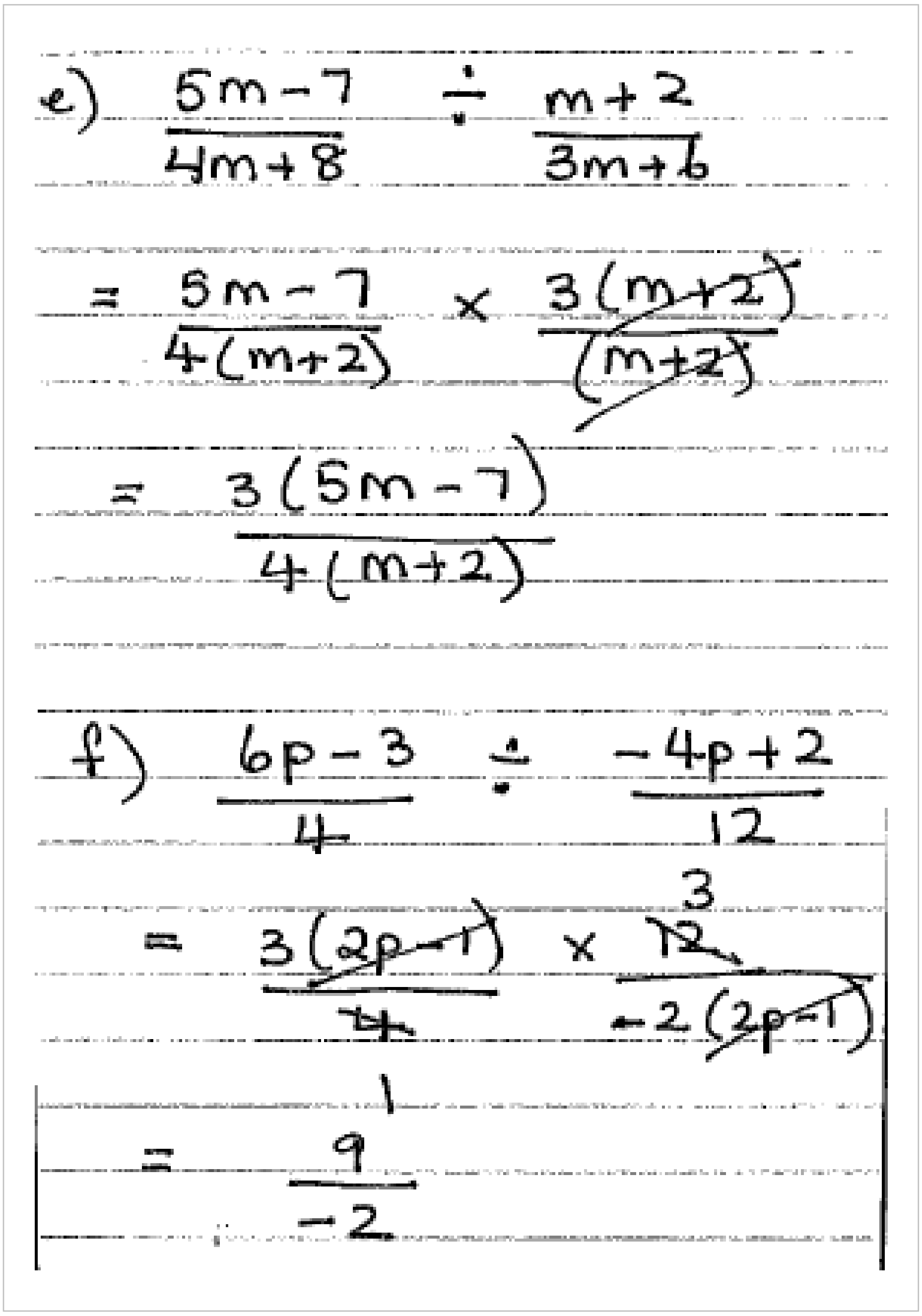
Factorises algebraic expressions correctly and expands binomial products.

...

Expands out brackets with negative numbers correctly.

Mathematics

Work sample 7:
Algebraic fractions



e) $\frac{5m-7}{4m+8} \div \frac{m+2}{3m+6}$

$= \frac{5m-7}{4(m+2)} \times \frac{3(m+2)}{(m+2)}$

$= \frac{3(5m-7)}{4(m+2)}$

f) $\frac{6p-3}{4} \div \frac{-4p+2}{12}$

$= \frac{3(2p-1)}{4} \times \frac{12}{-2(2p-1)}$

$= \frac{9}{-2}$

Annotations

Factorises and cancels correctly.

Uses the inversion of the divisor to multiply out algebraic fraction correctly.

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