Mathematics

Year 10

Above satisfactory

WORK SAMPLE PORTFOLIO

Annotated work sample portfolios are provided to support implementation of the Foundation – Year 10 Australian Curriculum.

Each portfolio is an example of evidence of student learning in relation to the achievement standard. Three portfolios are available for each achievement standard, illustrating satisfactory, above satisfactory and below satisfactory student achievement. The set of portfolios assists teachers to make on-balance judgements about the quality of their students' achievement.

Each portfolio comprises a collection of students' work drawn from a range of assessment tasks. There is no pre-determined number of student work samples in a portfolio, nor are they sequenced in any particular order. Each work sample in the portfolio may vary in terms of how much student time was involved in undertaking the task or the degree of support provided by the teacher. The portfolios comprise authentic samples of student work and may contain errors such as spelling mistakes and other inaccuracies. Opinions expressed in student work are those of the student.

The portfolios have been selected, annotated and reviewed by classroom teachers and other curriculum experts. The portfolios will be reviewed over time.

ACARA acknowledges the contribution of Australian teachers in the development of these work sample portfolios.

THIS PORTFOLIO: YEAR 10 MATHEMATICS

This portfolio provides the following student work samples:

Sample 1	Algebra: Heptathlon scoring
Sample 2	Statistics: Statistical logic
Sample 3	Probability: Probability and Venn diagrams
Sample 4	Measurement: Trigonometry – why not?
Sample 5	Geometry: Similar or congruent?
Sample 6	Measurement and statistics: How thirsty can you get?
Sample 7	Algebra and geometry: Quadratic equations
Sample 8	Algebra: Simultaneous equations
Sample 9	Geometry: Numerical exercises in geometry
Sample 10	Statistics: Quartiles
Sample 11	Algebra, measurement, geometry and statistics: Mathematics assignment

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Mathematics

Year 10
Above satisfactory

This portfolio of student work shows connections between algebraic and graphical representations of relations (WS11). The student solves surface area and volume problems relating to prisms and cylinders (WS6). The student finds unknown values after substitution into formulas (WS1, WS7, WS11), solves pairs of simultaneous equations (WS8) and solves quadratic equations (WS7, WS11). The student applies deductive reasoning to proofs and numerical exercises involving plane shapes (WS9, WS11). The student compares data sets (WS11) and investigates bivariate data where the independent variable is time (WS6). The student describes the relationship between two continuous variables (WS11) and evaluates statistical reports (WS2, WS7). The student calculates quartiles and inter-quartile ranges from a variety of data displays (WS10). The student uses triangle and angle properties to prove congruence and similarity (WS5) and explains how trigonometry can be used to calculate unknown sides and angles in right-angled triangles (WS4). The student lists outcomes for multi-step chance experiments and assigns probabilities for these experiments (WS3).

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Algebra: Heptathlon scoring

Year 10 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task/s are highlighted.

> *By the end of Year 10, students recognise the connection between simple and compound interest. They solve problems involving linear equations and inequalities. They make the connections between algebraic and graphical representations of relations. Students solve surface area and volume problems relating to composite solids. They recognise the relationships between parallel and perpendicular lines. Students apply deductive reasoning to proofs and numerical exercises involving plane shapes. They compare data sets by referring to the shapes of the various data displays. They describe bivariate data where the independent variable is time. Students describe statistical relationships between two continuous variables. They evaluate statistical reports.*
>
> *Students expand binomial expressions and factorise monic quadratic expressions. They find unknown values after substitution into formulas. They perform the four operations with simple algebraic fractions. Students solve simple quadratic equations and pairs of simultaneous equations. They use triangle and angle properties to prove congruence and similarity. Students use trigonometry to calculate unknown angles in right-angled triangles. Students list outcomes for multi-step chance experiments and assign probabilities for these experiments. They calculate quartiles and inter-quartile ranges.*

Summary of task

Students had been practising their algebraic skills. They were interested in the results from the athletics carnival and questioned how the heptathlon was scored. They were given this task to complete in class to demonstrate how well they could apply their algebraic skills to a relevant context.

Algebra: Heptathlon scoring

12. Heptathlon Scoring

In the Olympics, "all-round" women athletes compete in the Heptathlon. Held over two days, the athletes compete in the following events:
Day 1— 100 metres hurdles, high jump, shot put, and 200 metres
Day 2— long jump, javelin, and 800 metres.

Points are awarded for each event, and the athlete with the greatest total points is declared the winner. But how do they decide how many points to award to a particular performance, and how can you compare events? For example, how do you compare a 25 second performance in the 200 metre event with throwing the discus 75 metres?

Mathematics, Physics, and Computer Modelling were used by Dr Karl Ulbrich to create the current scoring system, which attempts to make fair comparisons between events.

There are three main rules used for calculating points in the seven events:

- The track events (200 m; 800 m; and 100 m hurdles): $P = a \times (b - T)^c$
- The jump events (high jump and long jump): $P = a \times (M - b)^c$
- The throwing events (shot put and javelin): $P = a \times (D - b)^c$

In these rules, P is the point score; T is the time in seconds; M is measurement in cm; and D is the distance in metres. a, b, and c are different for each event, as shown in the table:

EVENT	a	b	c
200 m	4.99	42.5	1.81
800 m	0.11	254	1.88
100 m hurdles	9.23	26.7	1.84
high jump	1.85	75.0	1.35
long jump	0.19	210	1.41
shot put	56.0	1.50	1.05
javelin	16.0	3.80	1.04

For example, a 29.30 second performance in the 200 metre event, would give the following number of points:

$$P = 4.99 \times (42.5 - 29.3)^{1.81}$$
$$= 532.6$$

Annotations

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Algebra: Heptathlon scoring

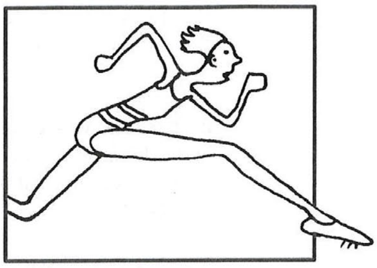
1. Write the appropriate rule to work out the points in each of the following cases, and then work out the points, using your calculator:

- a 32-second performance in the 200 m

$P = a(b-T)^c$

$P = 4.99(42.5-32)^{1.81}$

$P = 351.93$ (2dp)

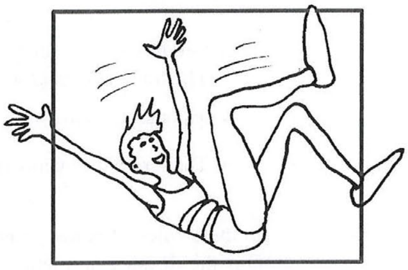


- a high jump of 1.80 m

$P = a(M-b)^c$

$P = 1.85(180-75)^{1.35}$

$P = 990.32$ (2dp)

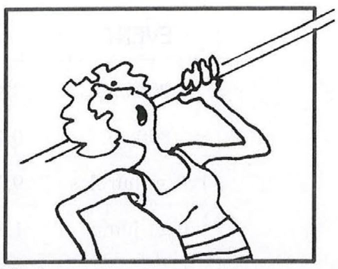


- a javelin throw of 65 m

$P = a(D-b)^c$

$P = 16(65-3.8)^{1.04}$

$P = 1154.36$ (2dp)



2. Using whatever method you think appropriate, find

- the time a runner would need to score 1000 points in the 200 m.

$1000 = a(b-T)^c$

$1000 = 4.99(42.5-T)^{1.81}$

$200.4 = (42.5-T)^{1.81}$

$18.69677469 = 42.5 - T$

$-23.80322531 = -T$

$\therefore T = 23.80$ seconds (2dp)

Annotations

Substitutes values from problem and table correctly into given formula and then calculates points.

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Algebra: Heptathlon scoring

- the distance in the long jump which would score the same number of points as a time of 1:55 in the 800 metre event.

$P = a(b-T)^c$

$P = 0.11(254-115)^{1.88}$

$P = 1175.60$ (2dp)

$1175.60 = a(M-b)^c$

$1175.60 = 0.19(M-210)^{1.41}$

$6187.368421 = (M-210)^{1.41}$

$488.6651085 = M-210$

$M = 698.67$ cm

3. At the 1988 Seoul Olympics, just before the last event (the 800 m), Jackie Joyner-Kersee of the USA needed 894 points to break the world record.

- What was the slowest time which Jackie could run and still break the world record?

$894 = a(b-T)^c$

$894 = 0.11(254-T)^{1.88}$

$8127.27 = (254-T)^{1.88}$

$120 = 254-T$

$-T = -133.8405615$

$T = 133.84$ sec

[Incidentally, Jackie ran 2:08.51, giving her a final total of 7291 points—still the current world record]

4. Now make up your own challenging question about the Heptathlon which can be answered using the information you have been given about the scoring system, and provide the calculations and the answer.

A high-jumper cleared the bar at 1.95m. A javelin thrower threw the javelin 70m. Who got the most points.

High Jump

$P = a(M-b)^c$

$P = 1.85(195-75)^{1.35}$

$P = 1185.95$ (2dp)

Javelin

$P = a(D-b)^c$

$P = 16(70-3.8)^{1.04}$

$P = 1252.60$ (2dp)

The Javelin thrower got 66.65 (2dp) more points

Annotations

Constructs a question and completes an answer with appropriate working.

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Statistics: Statistical logic

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Summary of task

Students had spent some time looking at media reports of statistical data. This task was given as a 10-minute test to evaluate how students could discern the facts from some statements.

Statistics: Statistical logic

Statistical Logic

Please comment on the three following uses of statistics. Does the logic in the statement make sense? Please explain your reasoning.

- "Young people account for 30% of all road accidents. Of course, this means that older drivers account for 70% of all road accidents- many more. The older drivers should get off the road and leave it to us young ones!"

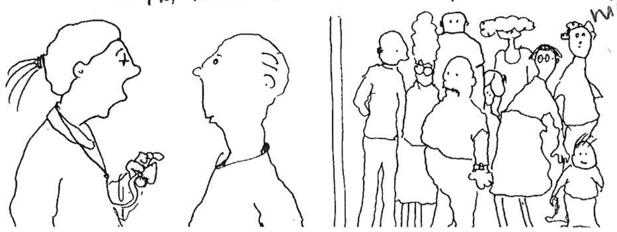
The reason the statistic is alarming is because 99% of people on the roads are old (estimate). Yet the 1% of young peope contribute to 30% of crashes. Depends on what you determine as old.

- "What is happening to our school system? It's a disgrace-50% of our students are below the school average!"

An average is made to have 50% higher and 50% lower. Even in our maths 50% of students are below average.

- A doctor informed a patient that he had a life-threatening disease, for which about 9 out of 10 patients usually died. "The good news", the doctor said, "is that my last nine patients have all died!"

The good news is … nine patients have already died. This does not mean that the next patient is safe and will live.



Annotations

Demonstrates sound understanding of the statistical fallacy in the logic in the quote and is able to explain the error clearly and succinctly using statistics.

Demonstrates an understanding of the average as a measure of the centre of the data and interprets the quote in another original example to further illustrate the fallacy in the argument.

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Probability: Probability and Venn diagrams

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Summary of task

Students had completed a unit of work on probability. They had spent several lessons applying their knowledge to experiments, recording results and calculating probabilities. Students were encouraged to reason through some problems and justify their conclusions using mathematical language. This task was given as a test during class time.

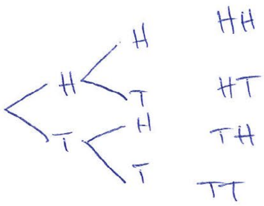
Mathematics

Year 10
Above satisfactory

Probability: Probability and Venn diagrams

Knowledge and Understanding
Question 1

a) Use a tree diagram to show the sample space for tossing 2 coins **simultaneously**.



b) Determine the probability of obtaining a head and a tail

p(H and a tail)
$= p(HT) + p(TH)$
$= \frac{1}{4} + \frac{1}{4}$
$= \frac{1}{2}$

c) Determine the probability of obtaining at least 1 head.

p(at least one head)
$= 1 - p(0 \text{ heads})$
$= 1 - \frac{1}{4}$
$= \frac{3}{4}$

Question 2

An equal- sector spinner containing numbers 1, 2, 3, and 4 is spun and an unbiased coin is tossed.

a) Draw a two way table to represent the sample space.

	1	2	3	4
H	H1	H2	H3	H4
T	T1	T2	T3	T4

b) Determine the probability of obtaining a head and a 4

p(head and a 4) $= \frac{1}{8}$

c) Determine the probability of obtaining a tail or an even number

p(a tail or an even number)
$= \frac{6}{8}$
$= \frac{3}{4}$

Annotations

Draws the two-way table with relevant information.

Demonstrates understanding of the tree diagram to represent the information from the experiment.

Interprets table correctly to calculate probability of event described.

Calculates probabilities correctly.

Deomonstrates an understanding of the concept of 'at least' to calculate the probability.

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Probability: Probability and Venn diagrams

Question 3

A box contains 3 red, 2 blue and 1 yellow marble. Two marbles are drawn **simultaneously** from the box. Determine the probability that they will be:

a) both red

$p(\text{both Red}) = \frac{3}{6} \times \frac{2}{5}$

$= \frac{6}{30}$

$= \frac{1}{5}$

b) red and blue

$p(\text{Red and Blue})$

$= \frac{3}{6} \times \frac{2}{5} \times 2$

$= \frac{12}{30}$

$= \frac{2}{5}$

c) both the same colour

$p(\text{same colour})$

$= P(RR) + p(BB)$

$= \frac{3}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{1}{5}$

$= \frac{6}{30} + \frac{2}{30}$

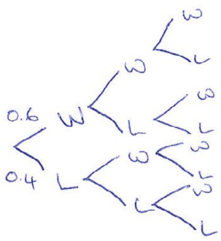
$= \frac{8}{30}$

$= \frac{4}{15}$

Question 4

In a chess tournament each contestant must play three others. Su estimates that she has a 60% chance of winning a game.

a) Draw a tree diagram and list the sample space for the possible outcomes of Su's three games.



WWW
WWL
WLW
WLL
LWW
LWL
LLW
LLL

b) Find the probability of Su winning at least 2 games.

$p(\text{at least 2 games})$

$= p(WWW) + p(WWL) + p(WLW) + p(L$

$= (0.6)^3 + (0.6)^2(0.4) \times 3$

$= \frac{81}{125}$

Annotations

Calculate the probabilities when the number of red marbles and the total number of marbles reduces by 1.

Constructs a tree diagram with correct values for the probability of each event occurring.

Demonstrates understanding of the words 'at least' and completes the calculation.

Realises that two red marbles or two blue marbles are the two possible solution outcomes.

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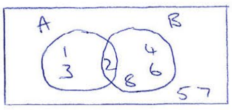
Probability: Probability and Venn diagrams

Question 5

An eight-sided die is rolled with faces numbered 1 – 8.

A is the event 'numbers less than 4' and B is the event 'numbers that are multiples of 2'.

a) Draw a Venn diagram to represent the above information.



b) Calculate the probability that the number is less than 4 given that it is a multiple of 2.

p(no is less than 4, given its a multiple of 2)

$= \frac{1}{4}$

c) Determine that probability that the number is a multiple of 2 given it is less than 4.

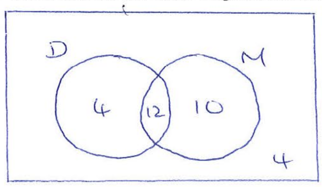
p(no. is a multiple of 2, given it is less than 4)

$= \frac{1}{3}$

Question 6

A teacher surveys her class of students about their chocolate preferences. Out of 30 students, 16 students liked dark chocolate (D) and 22 students liked milk chocolate (M). Only 4 of the students surveyed liked neither milk nor dark chocolate.

(a) Show this on the Venn diagram below.



Let x = no. of students who like both D and M

$16 - x + x + 22 - x + 4 = 30$

$42 + x = 30$

$x = 12$

(b) Determine the probability that a randomly selected student from the class likes:

(i) both milk and dark chocolate

p(both M + D) $= \frac{12}{30}$

$= \frac{2}{5}$

(ii) milk chocolate only

p(m only) $= \frac{10}{30}$

$= \frac{1}{3}$

Annotations

Represents the information on a Venn diagram.

Determines the intersection of the two sets and hence calculates the value contained in the intersection using algebraic techniques.

Determines the intersection of the two sets to answer the question.

Calculates the probability from information contained in the Venn diagram.

Recognises the difference between the order of the requirements contained in question 5. c), and hence the difference in the answer from question 5. b).

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Measurement: Trigonometry – why not?

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Summary of task

Students had been investigating trigonometry. They had looked at the applications and use of trigonometry. Students were asked to complete these questions as a formative assessment task to give the teacher an indication of how much revision the student required.

Measurement: Trigonometry – why not?

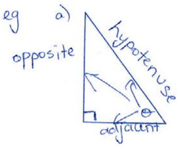
Trigonometry- Why not?

A person was quoted in the local paper as saying 'the things you learn at school are just not relevant when you leave school'. My Maths teacher was just horrified and asked the following questions:

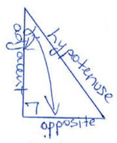
1. Write down all that you know about the trigonometric ratios.
2. Why would people need these ratios in life outside school?

1. Trigonometry is the study of how the sides and angles of a right angled triangle are related to each other

eg a)



b)



The hypotenuse is always the side opposite the right angle.

The adjacent side is always the side next to the angle and is one of the arms of the angle

The opposite side is always directly opposite the angle.

There are 3 ratios which link the identified angle and the sides

① The sine ratio = $\frac{\text{side opposite}}{\text{hypotenuse}}$

② The cosine ratio = $\frac{\text{side adjacent}}{\text{hypotenuse}}$

③ The tangent ratio = $\frac{\text{side opposite}}{\text{side adjacent}}$

These are abbreviated to:-

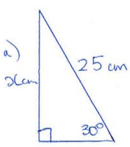
① $\sin\theta = \frac{opp}{hyp}$

② $\cos\theta = \frac{adj}{hyp}$

③ $\tan\theta = \frac{opp}{adj}$.

Using these ratios you can calculate an unknown side or an unknown angle of any right angled triangle

eg a)



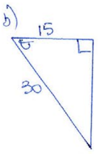
$\sin 30° = \frac{x}{25}$

$x = 25 \times \sin 30°$

$= 25 \times 0.5$

$= 12.5\text{cm}$

b)



$\cos\theta = \frac{15}{30}$

$\cos\theta = \frac{1}{2} = 0.5$

$\theta = \cos^{-1} 0.5$

$\theta = 60°$

You can use the trigonometric ratios for any sized triangle because the ratios would stay the same because of similarity no matter how large the triangle is.

2. People would use these trigonometric ratios in many various ways especially in the building and construction areas. You can use this to find the heights of towers buildings or mountains, navigation or space travel. It is one of the most commonly used bits of mathematics. Architects use trigonometry in every part of their work including analysing sun shading and light aspects of buildings

Annotations

Explains the three ratios and the relationship between angles and sides.

Gives examples of how to calculate a side and an angle.

Acknowledges the relationship between similarity and the trigonometric ratios.

Explains the usefulness of trigonometry in a variety of contexts.

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Geometry: Similar or congruent?

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Students expand binomial expressions and factorise monic quadratic expressions. They find unknown values after substitution into formulas. They perform the four operations with simple algebraic fractions. Students solve simple quadratic equations and pairs of simultaneous equations. They use triangle and angle properties to prove congruence and similarity. Students use trigonometry to calculate unknown angles in right-angled triangles. Students list outcomes for multi-step chance experiments and assign probabilities for these experiments. They calculate quartiles and inter-quartile ranges.

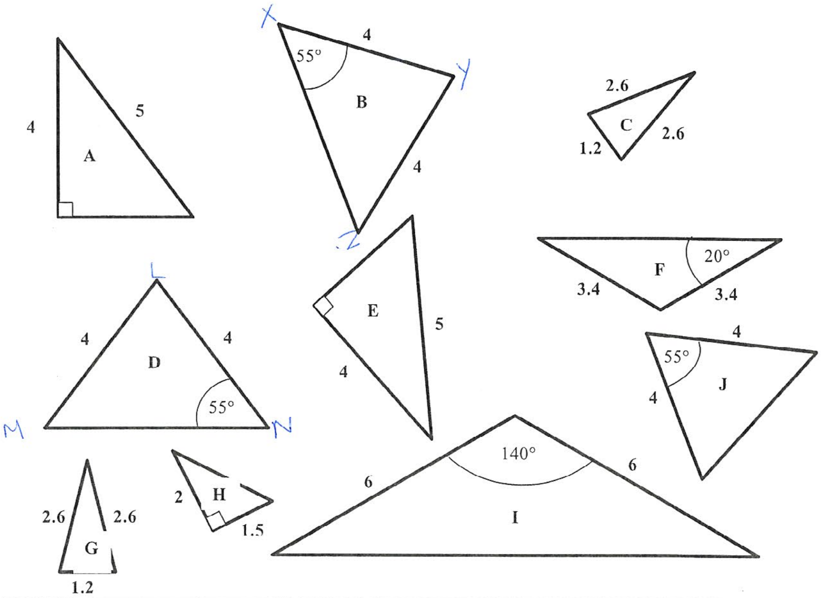
Summary of task

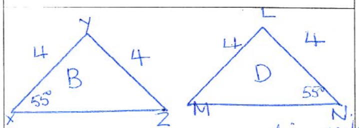
Students had studied both similarity and congruence during the year. They were asked to make connections between the two concepts and to complete a task which involved both concepts. The teacher wanted to ensure that students could clearly identify the difference between similarity and congruence.

Geometry: Similar or congruent?

Similar? Congruent?

1. Consider the following triangles and complete the table below. (All lengths are in centimetres)



Which triangles are similar?	Which triangles are congruent?	Reasons for congruency
$A \approx H \approx E$ $C \approx G$ $I \approx F$	$B \equiv D$	 $\angle Y = \angle L$ (corresponding angles of isosceles triangle) $= 70°$ $XY = LM$ (given) $YZ = LN$ (given) $\therefore \triangle XYZ \equiv \triangle LMN$ (SAS test)

Annotations

Identifies the similar triangles using the correct symbol for similarity.

Identifies the congruent triangles using the correct symbol for congruence.

Uses reasoning to demonstrate why triangles are congruent.

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Measurement and statistics: How thirsty can you get?

Year 10 Mathematics achievement standard

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Students expand binomial expressions and factorise monic quadratic expressions. They find unknown values after substitution into formulas. They perform the four operations with simple algebraic fractions. Students solve simple quadratic equations and pairs of simultaneous equations. They use triangle and angle properties to prove congruence and similarity. Students use trigonometry to calculate unknown angles in right-angled triangles. Students list outcomes for multi-step chance experiments and assign probabilities for these experiments. They calculate quartiles and inter-quartile ranges.

Summary of task

Students had spent two weeks investigating surface area and volume. They were given this task as an assignment to apply the skills they had learnt in class to a real-world problem. They were asked to solve a problem using their knowledge of surface area and volume to perform calculations and justify their results.

Measurement and statistics: How thirsty can you get?

Task 1

months	water supply	water usage	water over	water in tank	
JAN	195 x 0.2 = 39	.5934 x 31 = 18.4	20.6	20.6	
FEB	200 x 0.2 = 40	.5934 x 29 = 17.2	22.8	43.4	
MAR	150 x 0.2 = 30	.5934 x 31 = 18.4	12.6	56	
APR	80 x 0.2 = 16	.5934 x 30 = 17.8	-1.8	54.2	
MAY	55 x 0.2 = 11	.5934 x 31 = 18.4	-7.4	46.8	
JUN	35 x 0.2 = 7	.5934 x 30 = 17.8	-10.8	36	
JUL	30 x 0.2 = 6	.5934 x 31 = 18.4	-12.4	23.6	
AUG	20 x 0.2 = 4	.5934 x 31 = 18.4	-14.4	9.2	
SEP	28 x 0.2 = 5.6	.5934 x 30 = 17.8	-12.2	-3	buy water
OCT	65 x 0.2 = 13	.5934 x 31 = 18.4	-5.4	-5.4	buy water
NOV	69 x 0.2 = 13.8	.5934 x 30 = 17.8	-4	-4	buy water
DEC	175 x 0.2 = 35	.5934 x 31 = 18.4	16.6	16.6	

Annotations

Demonstrates an excellent understanding of a real-world problem and gives a sound solution based on their calculations.

Uses a table to correctly compare four variables (water available, water used, 'water over' and the amount of water that remains in the tank) against time in the first year.

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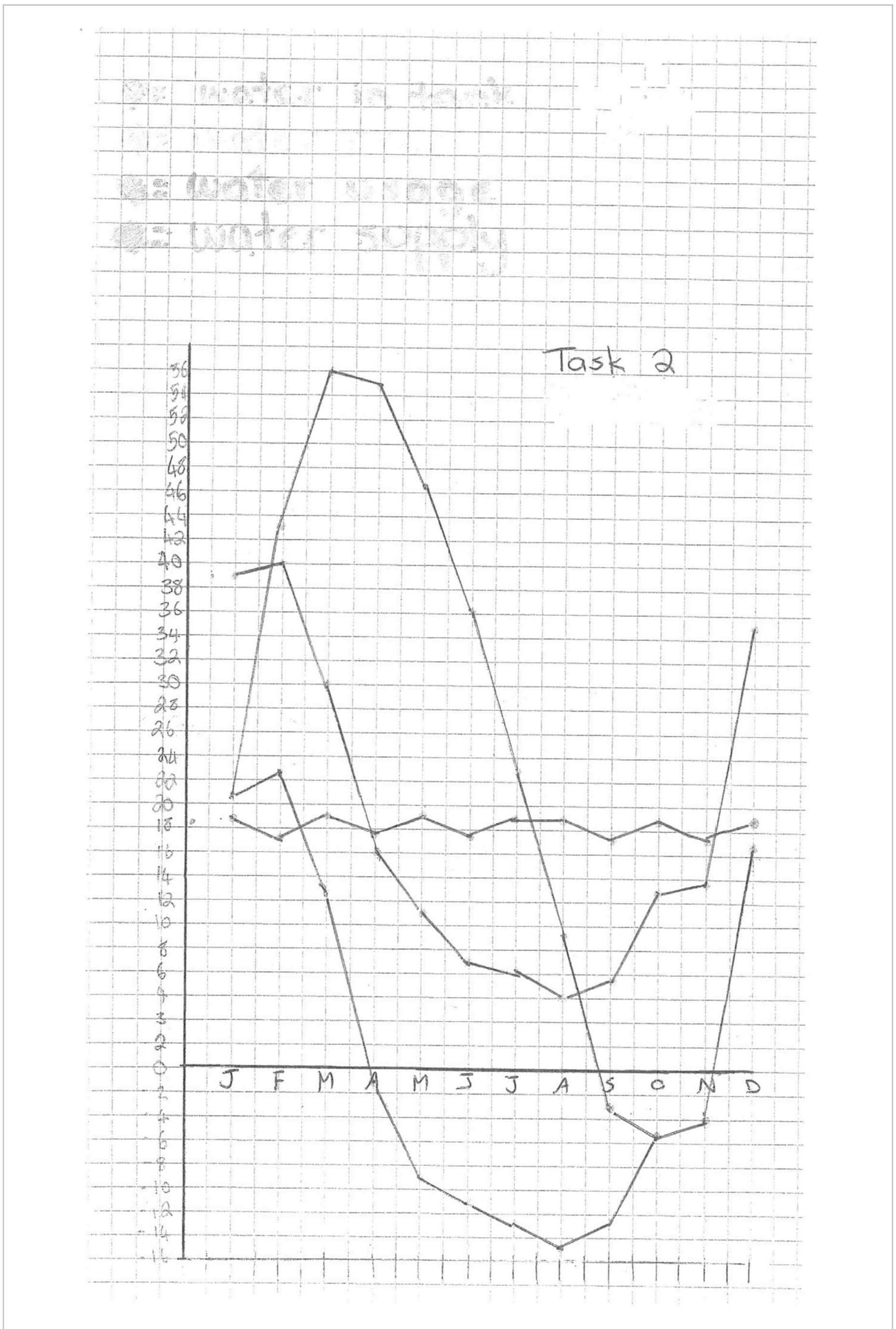
Measurement and statistics: How thirsty can you get?



Annotations

Graphs water supply, water usage, water over and water in the tank against the independent variable, time, on the same axes.

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Measurement and statistics: How thirsty can you get?

Task 3

I think that an appropriate tank size for this farm is $60 m^3$. This tank will not overflow as the highest amount of water over the year is 56 so there will not be any overflow, but there is not a lot of wasted area either.

Task 4

The $60 m^3$ tank costs $1800 from "Tanks r us". We also have to purchase water for September, October and November (12.4 kL)

Total cost will be $1800 + $124 = $1924

Annotations

Clearly communicates the reason for the choice of size.

Correctly calculates the cost of the suggested tank in the first year using the table of water supply/water consumption.

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Measurement and statistics: How thirsty can you get?

Task 5

months	Water supply	Water usage	Water over	Water in tank	
JAN	195 × 0.2 = 39	.5934 × 31 = 18.4	20.6	37.2	
FEB	200 × 0.2 = 40	.5934 × 28 = 16.6	23.4	60	
MAR	150 × 0.2 = 30	.5934 × 31 = 18.4	12.6	60	
APR	80 × 0.2 = 16	.5934 × 30 = 17.8	-1.8	58.2	
MAY	55 × 0.2 = 11	.5934 × 31 = 18.4	-7.4	50.8	
JUN	35 × 0.2 = 7	.5934 × 30 = 17.8	-10.8	40	
JUL	30 × 0.2 = 6	.5934 × 31 = 18.4	-12.4	27.6	
AUG	20 × 0.2 = 4	.5934 × 31 = 18.4	-14.4	13.2	
SEP	28 × 0.2 = 5.6	.5934 × 30 = 17.8	-12.2	1	
OCT	65 × 0.2 = 13	.5934 × 31 = 18.4	-5.4	-4.4	buy water
NOV	69 × 0.2 = 13.8	.5934 × 30 = 17.8	-4	+4	buy water
DEC	175 × 0.2 = 35	.5934 × 31 = 18.4	16.6	16.6	

In the second year of operation it will cost $84 in water.

Task 6

Some alternative tank sizes are 40m³ and 100m³

40m³ is too small as it would overflow a few times through the year, wasting water.
100m³ is a waste of space and money as there will never be enough water rainfall.

Annotations

Correctly completes calculations for each variable throughout the second year, taking in to account the amount of water remaining in the tank at the end of the first year.

Summarises the results from the table.

Communicates reasons why two alternative tank sizes are less suitable than the chosen size.

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Mathematics

Year 10
Above satisfactory

Measurement and statistics: How thirsty can you get?

Cylinder

Size $64 m^2$

$- 2r = h$

$TSA = 2\pi r(r+h)$

$= 2\pi r(r+2r)$

$= 2\pi r \times 3r$

$= 6\pi r^2$

$= \sqrt{\frac{64}{6\pi}}$

$r = 1.84$

$h = 3.68$

Vol Task 7

$= \pi r^2 \times h$

$= 3.14 \times 1.84^2 \times$ [illegible]

$= 39 m^3$

Size $85 m^2$

$- 2r = h$

$TSA = 2\pi r(r+2r)$

$= 2\pi r \times 3r$

$= 6\pi r^2$

$= \sqrt{\frac{85}{6\pi}}$

$r = 2.12$

$h = 4.24$

VOL

$= \pi r^2 \times h$

$= 3.14 \times 2.12^2 \times 4.24$

$= 59 m^3$

Size 120

$- 2r = h$

$TSA = 2\pi r(r+h$

$= 2\pi r(r+2r)$

$= 2\pi r \times 3r$

$= 6\pi r^2$

$= \sqrt{\frac{120}{6\pi}}$

$r = 2.52$

$h = 5.04$

VOL

$= \pi r^2 \times h$

$= 3.14 \times 2.54^2 \times 5.04$

$= 102 m^3$

Annotations

Calculates the volumes of three different cylinders from the given surface areas and using a clearly stated assumption that the height is twice the length of the radius.

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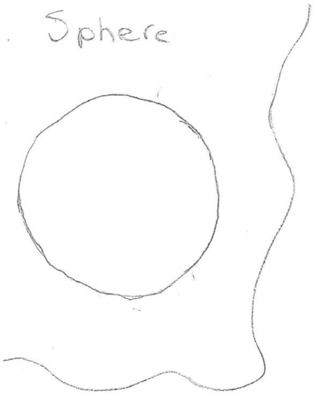
Mathematics

Year 10

Above satisfactory

Measurement and statistics: How thirsty can you get?

Sphere



Size 64

Task 7

$TSA = 4\pi r^2$

$64 = 4\pi r^2$

$= \sqrt{\frac{64}{4\pi}}$

$r = 2.26$

VOL

$= \frac{4}{3} \times 3.14 \times (2.26)^3$

$= 48 m3$

Size 85

$TSA = 4\pi r^2$

$85 = 4\pi r^2$

$= \sqrt{\frac{85}{4\pi}}$

$r = 2.6$

VOL

$= \frac{4}{3} \times 3.14 \times (2.6)^3$

$= 73 m3$

Size 120

$TSA = 4\pi r^2$

$120 = 4\pi r^2$

$= \sqrt{\frac{120}{4\pi}}$

$r = 6.8$

VOL

$= \frac{4}{3} \times 3.14 \times (6.8)^3$

$= 1,316 m^3$

Annotations

Chooses to compare the volumes of cylinders with spheres and cubes, recognising that both of these solids require the use of only one unknown in the calculation.

Calculates the volumes of three different spheres from the given surface areas.

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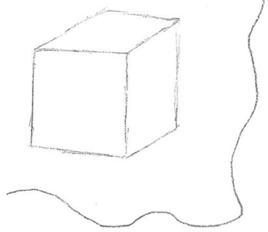
Measurement and statistics: How thirsty can you get?

Cube



Size $64m^2$

Task 7

$TSA = 64$

$6x^2 = 64$

$x = \sqrt{\frac{64}{6}}$

$x = 3.3$

VOL

$= 3.3^3$

$= 35m^3$

Size 85

$TSA = 85$

$6x^2 = 85$

$x = \sqrt{\frac{85}{6}}$

$x = 3.8$

VOL

$= 3.8^3$

$= 68m^3$

Size 120

$TSA = 120$

$6x^2 = 120$

$x = \sqrt{\frac{120}{6}}$

$x = 4.5$

VOL

$= 4.3^3$

$= 91m^3$

Annotations

Calculates the volumes of three different cubes from the given surface areas.

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Measurement and statistics: How thirsty can you get?

Task 8

I have found that the best option for size and volume is an $85 m^2$ Sphere, this is not a valid option as it isn't practical because of the shape.

The next best option for price and volume is the $85 m^2$ Cylinder. It cost \$300 less to buy the same size tank from "TANKS 4 YOUR LOOT" over "TANKS R US".

I have chosen this tank due to volume, cost and practicality

Annotations

Determines the best practical solution to the problem with appropriate reasoning referring to the volume, cost and practicality of a range of different-shaped solids.

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Algebra and geometry: Quadratic equations

Year 10 Mathematics achievement standard

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Summary of task

Students had spent some time solving quadratic equations and solving problems that required them to form a quadratic equation as a way to find a solution to a problem. This task was set as a class test that took 20 minutes.

Mathematics

Year 10
Above satisfactory

Algebra and geometry: Quadratic equations

Year 10 Quadratic Equations work sample #

1 Which of the options given are the solution(s) to each of these equations?
*(Circle **all** that apply.)*

(a) $3y + 7 = 92 - 2y$

A 6 **B** 7 **C** −6 **D** −7 **E** none of the above

(b) $x^2 - 24 = 5x$

A 8 **B** 12 **C** −2 **D** −3 **E** none of the above

(c) $m^2 = -100$

A 5 **B** −10 **C** 10 **D** −50 **E** none of the above

(d) $x^3 - 2x^2 - 11x + 12 = 0$

A −4 **B** 4 **C** −3 **D** 1 **E** none of the above

2 Provide **exact** solutions (i.e. $\sqrt{5}$, not 2.236) to the following equations.

a $y^2 = 4$

$y = \pm 2$

b $x^2 - 21 = 0$

$x^2 = 21$

$x = \pm\sqrt{21}$

c $\frac{2x^2 + 7}{3} = 100$

$2x^2 + 7 = 300$

$2x^2 = 293$

$x^2 = \frac{293}{2}$

$x = \pm\sqrt{\frac{293}{2}}$

d $(a + 4)(a - 1) = 0$

$a = -4, 1$

e $6(2m - 1)(3m + 4) = 0$

$(2m - 1)(3m + 4) = 0$

$m = \frac{1}{2}, -\frac{4}{3}$

Annotations

Selects the correct option from the given options.

Demonstrates an understanding that quadratic equations can have two solutions and correctly determines the two solutions.

Recognises that some quadratic equations cannot be solved for real solutions.

Correctly solves simple quadratic equations, demonstrating an understanding of the concept of an exact solution.

Demonstrates a correct procedure for solving a simple quadratic equation involving a fraction, leaving the answer in simplest exact form.

Demonstrates knowledge of how to solve quadratic equations given in factored form.

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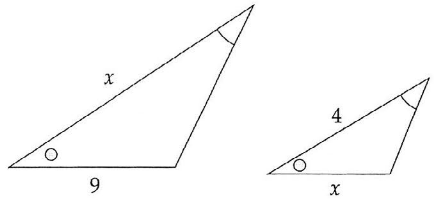
Algebra and geometry: Quadratic equations

3 In the following diagram, the two triangles are similar.

Write an appropriate equation and solve it to find the value of x.



For triangles to be similar then sides are in the same ratio

$\therefore \quad 9 : x = x : 4$

$\frac{9}{x} = \frac{x}{4}$

$36 = x^2$

$x = \pm 6$

But x cannot be negative as you can't have a negative length

$\therefore \quad x = 6$

Annotations

Applies knowledge of similar figures to form equivalent ratios.

Converts the equivalent ratios into a quadratic equation that allows efficient calculation of the solution.

Considers the reasonableness of the mathematical solution for the given context and justifies why one of the obtained solutions is not appropriate.

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Algebra: Simultaneous equations

Year 10 Mathematics achievement standard

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Summary of task

Students completed a unit of work on equations. The unit included looking at different methods of solving linear and simultaneous equations, including applying these techniques to solve word problems. The students were given 20 minutes to complete this assessment task.

Algebra: Simultaneous equations

1 How many solutions does the equation $7x + 5y = 24$ have? Explain.

$7x+5y-24=0$

$\therefore$ 2 solutions

2 Solve $2x - y = 5$ if:

(i) $x = 5$

$2(5)-y=5$

$10-5=y$

$y=5$

(ii) $y = -2$

$2x-(-2)=5$

$2x+4=5$

$2x=1$

$x=\frac{1}{2}$

3 Solve the following equations simultaneously:

(i) $3x + y = 10$ and $x - y = -2$ ②

$x=-2+y$ —①

$3(-2+y)+y=10$

$-6+3y+y=10$

$4y=16$

$y=4$

put $y=10$ into ①

$x=-2+10$

$=8$

$\therefore$ the solution is

$x=8,\ y=4$

(ii) $2x + 9y = 43$ ② and $y = x - 1$ ①

$2x+9(x-1)=43$

$2x+9x-9=43$

$11x=52$

$x=\frac{52}{11}$

put $x=\frac{52}{11}$ into ①

$y=\frac{52}{11}-1$

$=\frac{41}{11}$

$\therefore$ The solution is $x=\frac{52}{11}$ and $y=\frac{41}{11}$

Annotations

Attempts to interpret the question.

Finds the value of one variable given the value of the other.

Uses a correct technique to solve the equation but makes a minor calculation error.

Demonstrates understanding of the substitution method when solving pairs of simultaneous equations, but makes a minor error.

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Mathematics

Year 10

Above satisfactory

Algebra: Simultaneous equations

3 *(continued)*

(iii) $7x - 3y = -20$ and $3x + 5y = 31$ — ①

$x = \frac{-20+3y}{7}$ — ②

$3\left(\frac{-20+3y}{7}\right) + 5y = 31$

$\frac{-60+9y}{7} + 5y = 31$

$-60 + 9y + 35y = 217$

$44y = 277$

$y = \frac{277}{44}$

put $y = \frac{277}{44}$ into ②

$x = \frac{-20+3\left(\frac{277}{44}\right)}{7}$

$= -\frac{7}{44}$

$\therefore$ The solution is $x = -\frac{7}{44}$ and $y = \frac{277}{44}$.

4 Choose **one** of the problems below and use simultaneous equations to solve it.

I A piggybank contains only 20c and 50c coins. There are 60 coins in total and their overall value is $18.60. Determine the contents of the piggybank.

II A man is currently six times older than his granddaughter. Thirteen years ago, he was 18 times older than her. What are their current ages?

I. Let 20c be x coins and 50c be y coins.

$x + y = 60$ — ①

$0.2x + 0.5y = 18.6$ — ②

sub ① into ②

$0.2(60 - y) + 0.5y = 18.6$

$12 - 0.2y + 0.5y = 18.6$

$0.3y = 6.6$

$y = 22$

sub $y = 22$ into ①

$x = 60 - 22$

$= 38$

$\therefore$ There are 38 20c coins and 22 50c coins.

Annotations

Demonstrates sound algebraic skills to accurately solve a pair of simultaneous equations.

Interprets a word problem, represents it mathematically and applies an appropriate algebraic technique to find the correct solution.

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Geometry: Numerical exercises in geometry

Year 10 Mathematics achievement standard

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Summary of task

Students had studied a unit of work on geometrical reasoning. An assessment task was given at the end of the unit. Students were expected to spend between 10 and 15 minutes to complete this task.

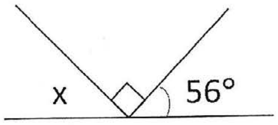
Mathematics

Year 10
Above satisfactory

Geometry: Numerical exercises in geometry

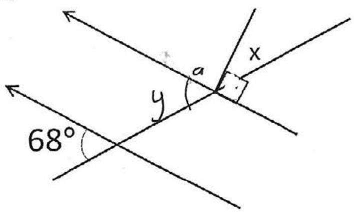
Calculate the values of the unknown angles x in each of the diagrams below.

a)



$x + 90 + 56 = 180°$
$x + 146 = 180$
$x = 180 - 146$
$x = 34°$

b)

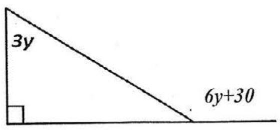


$y = 68°$

$a + 90 = 180$
$a = 180 - 90$
$a = 90°$

$y + a + x = 180°$
$68 + 90 + x = 180$
$158 + x = 180$
$x = 180 - 158$
$x = 22°$

Calculate the value of *y* in the following diagram.



$90 + 3y = 6y + 30$
$90 - 30 = 6y - 3y$
$\frac{60}{3} = \frac{3y}{3}$
$20° = y$

Annotations

Recognises the straight angle and establishes an equation to solve the problem.

Introduces additional variables to assist in solving the problem, applying equations to obtain correct values and showing all steps in their reasoning.

Uses an efficient approach to obtain the correct value by recognising that the exterior angle of a triangle is equal to the sum of the opposite two interior angles.

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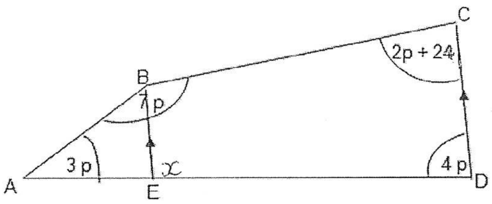
Geometry: Numerical exercises in geometry

(a) *Use algebraic methods to find the value of p.*

(b) *Determine the size of ∠ABE.*



a) $7p + 3p + 4p + 2p + 24 = 360°$

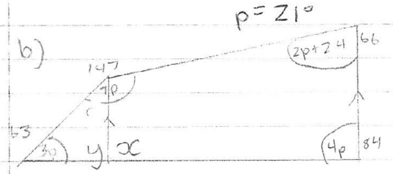
$16p + 24 = 360$

$16p = 360 - 24$

$\frac{16p}{16} = \frac{336}{16}$

$p = 21°$

b)



$x + 4p = 180°$

$x + 4 \times 21 = 180$

$x + 84 = 180$

$x = 180 - 84$

$x = 96°$

$x + y = 180°$

$96 + y = 180$

$y = 180 - 96$

$y = 84°$

$\angle ABE + \angle BEA + \angle EAB = 180°$

$\angle ABE + y + 3p = 180$

$\angle ABE + 84 + 3 \times 21 = 180$

$\angle ABE + 147 = 180$

$\angle ABE = 180 - 147$

$\angle ABE = 33°$

Annotations

Applies the angle sum of a quadrilateral to establish an equation and solve the problem.

Introduces additional variables to assist in solving the problem, applying equations to obtain correct values and showing all steps in their reasoning.

Uses geometrical notation to communicate reasoning and solve the problem.

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Mathematics

Year 10
Above satisfactory

Statistics: Quartiles

Year 10 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task/s are highlighted.

By the end of Year 10, students recognise the connection between simple and compound interest. They solve problems involving linear equations and inequalities. They make the connections between algebraic and graphical representations of relations. Students solve surface area and volume problems relating to composite solids. They recognise the relationships between parallel and perpendicular lines. Students apply deductive reasoning to proofs and numerical exercises involving plane shapes. They compare data sets by referring to the shapes of the various data displays. They describe bivariate data where the independent variable is time. Students describe statistical relationships between two continuous variables. They evaluate statistical reports.

Students expand binomial expressions and factorise monic quadratic expressions. They find unknown values after substitution into formulas. They perform the four operations with simple algebraic fractions. Students solve simple quadratic equations and pairs of simultaneous equations. They use triangle and angle properties to prove congruence and similarity. Students use trigonometry to calculate unknown angles in right-angled triangles. Students list outcomes for multi-step chance experiments and assign probabilities for these experiments. They calculate quartiles and inter-quartile ranges.

Summary of task

Students had spent some time studying statistics, including the calculation of quartiles and inter-quartile ranges in five-number summaries from a variety of data displays. This task was set for students to complete in 20 minutes of class time.

Statistics: Quartiles

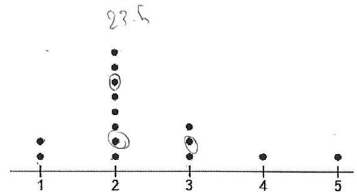
In this work sample, you will calculate the five-number summary for several data sets. The data sets are labelled **A–G**, and your answers are to be placed in the table in the middle of the page.

A 4 7 8 8 10 16 18 19 19 20 23 23 24
B 4 7 8 8 10 16 18 19 19 20 23 23 24 27
C 4 7 8 8 10 16 18 19 19 20 23 23 24 27 28
D 4 7 8 8 10 16 18 19 19 20 23 23 24 27 28 29

E

Score	Frequency	
1	8	8
2	11	19
3	15	34
4	2	36
5	9	45
6	15	60
7	18	78

F



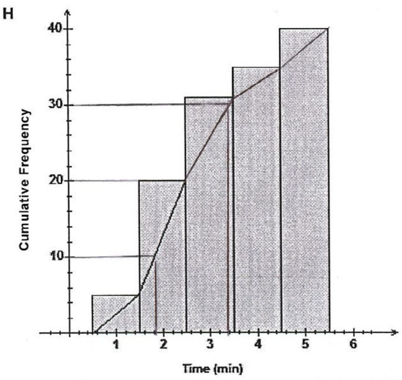
G

Stem	Leaf
0	3 4
0	8 9 9
1	2 2 3 4 4 4
1	5 6 7 7 7 8 8
2	0 1 1 1 2 2 3 4

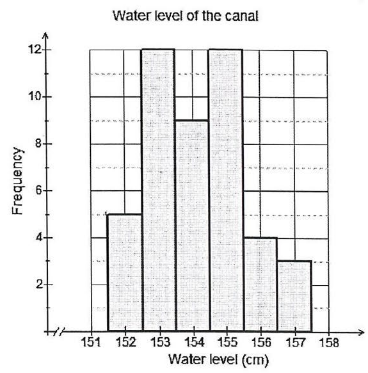
n = 26

	Min	Q1	Med	Q3	Max	IQR
A	4	8	18	21.5	24	13.5
B	4	8	18.5	23	27	15
C	4	8	19	23	28	15
D	4	9	19	23.5	29	14.5
E	1	3	5	6	7	3
F	1	2	2	3	5	1
G	3	12	16.5	21	24	9
H	1	1.8	2.5	3.4	5	1.6
I	152	153	154	155	157	2

H



I



n = 46

Annotations

Determines quartiles and inter-quartile ranges from ordered lists of data.

Determines quartiles and inter-quartile ranges from data displayed in dot plots and stem-and-leaf plots.

Determines the quartiles and inter-quartile range for continuous data displayed in a cumulative frequency histogram but does not accurately find the minimum and maximum values.

Determines the quartiles and inter-quartile range from data displayed in a frequency table and frequency histogram.

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Algebra, measurement, geometry and statistics: Mathematics assignment

Year 10 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task/s are highlighted.

By the end of Year 10, students recognise the connection between simple and compound interest. They solve problems involving linear equations and inequalities. They make the connections between algebraic and graphical representations of relations. Students solve surface area and volume problems relating to composite solids. They recognise the relationships between parallel and perpendicular lines. Students apply deductive reasoning to proofs and numerical exercises involving plane shapes. They compare data sets by referring to the shapes of the various data displays. They describe bivariate data where the independent variable is time. Students describe statistical relationships between two continuous variables. They evaluate statistical reports.

Students expand binomial expressions and factorise monic quadratic expressions. They find unknown values after substitution into formulas. They perform the four operations with simple algebraic fractions. Students solve simple quadratic equations and pairs of simultaneous equations. They use triangle and angle properties to prove congruence and similarity. Students use trigonometry to calculate unknown angles in right-angled triangles. Students list outcomes for multi-step chance experiments and assign probabilities for these experiments. They calculate quartiles and inter-quartile ranges.

Summary of task

This group assignment was completed at the end of a semester. It assessed several topics including quadratic equations, bivariate data, statistics and algebraic graphical representations.

In this assignment students collected data from an experiment. The assignment measured the student's understanding and the interrelationships of mathematical concepts and reasoning to draw conclusions based on the data. The students were given one week to complete the task.

Algebra, measurement, geometry and statistics: Mathematics assignment

Maths assignment- year 10

Part A

Knowledge and procedures

- ✓ Task 1: Create models of the whirlybird
- ✓ Task 2: Test fly whirlybirds detailing the conditions
- ✓ Task 3: Collect, record and summarise the data obtained in an appropriate form
- ✓ Task 4: Produce a scatter plot of the data

Task 1 was completed successfully as a team collaboration. Layouts and designs of the whirlybirds were provided to each student and the diagrams in Appendix 1 were followed to complete the models of the whirlybirds. Since the time a whirlybird is in flight may be effected by the length of the wings, the width of the wings and/or the size of its body tests were conducted to see if variations in length had any effect on how long the whirlybird was in flight for. Measurements were taken of the length of the whirlybird's wing and it measured to be 15.3cm in length therefore 3mm were cut off to make the measurement a round number. The length of the eight different whirlybird wing differed in sizes of two cm. Starting as small as a length of one cm and increasing to 15 cm. Once the layouts of the whirlybirds were cut out a paper clip was attached to the bottom of the folded whirlybird to keep the paper intact.

Task 2, the testing of the whirlybirds, was conducted in the same conditions and we the same variables were current in each test. The variables that were constant are listed below:

- The width of the wings on the whirlybird were the same
- The size of the whirlybirds body's were the same
- The same timer (stopwatch) was used
- The whirlybirds were thrown from the same height (220.5cm)
- The whirlybirds were thrown by the same person using the same technique every time
- The whirlybirds were in the same controlled environment (inside the classroom)
- The same person timed every whirlybird flight through the course of the experiment

Unfortunately, some variables may have affected by the time that the whirlybird was in flight for and these changing variables are listed below:

- The wing lengths of the whirlybirds were purposely changed to see if it had any effect on the time the whirlybirds were in flight for
- One side of the windows were open in the classroom and the conditions could most probably have changed throughout the course of the experiment. Since the speed of the wind isn't always constant
- Louise (in charge of throwing the whirlybirds) could have accidently applied more strength in her releases/throws for some of the whirlybirds. This would ultimately effect the time the whirlybird were in flight for

Tests to see how long the whirlybirds were in flight for were conducted three times per measurement which meant that an average result of how long the whirlybirds were in flight for

Annotations

Demonstrates the ability to design investigations and plan their approach to answering a question.

Interrogates their solutions and uses a number of different approaches to subject them to rigorous scrutiny, successfully verifying that their answers are reasonable.

Chooses appropriate methods and approximations.

Identifies a variety of variables that impact on the results of trials.

Presents meticulous records of trials and working.

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Algebra, measurement, geometry and statistics: Mathematics assignment

could be found. By finding an average in the results we could eliminate the possibility that the changing variables could have any effect on the flight time of each whirlybird.

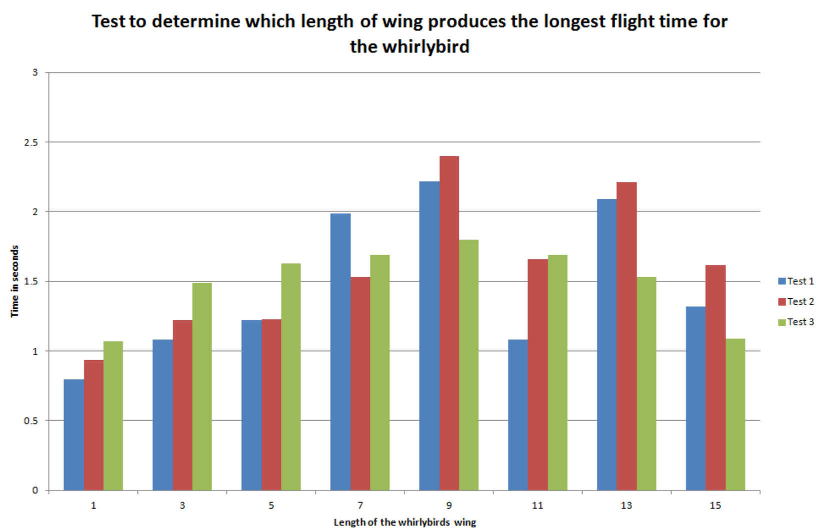
The table below is a collection of the data obtained during the experiment. The first column is the measurements in centimetres of the whirlybirds changing wing lengths. The next columns represent the three tests that were conducted. The total represents the addition of the three tests for each measurement of wing length. The average is found by adding the flight times from the three different tests and dividing that number by the number of tests conducted, which in this case was three. The table below shows the results from the experiment.

(Figure 1)

Cm	Test 1	Test 2	Test 3	Total	Average
1	0.8	0.94	1.07	2.81	0.936667
3	1.08	1.22	1.49	3.79	1.263333
5	1.22	1.23	1.63	4.08	1.36
7	1.99	1.53	1.69	5.21	1.736667
9	2.22	2.4	1.8	6.42	2.14
11	1.08	1.66	1.69	4.43	1.476667
13	2.09	2.21	1.53	5.83	1.943333
15	1.32	1.62	1.09	4.03	1.343333

The column graph below aims at demonstrating furthermore how the length of the wing of the whirlybirds could effect the results.

(Figure 2)



Annotations

Collects, records and graphs data.

Determines averages.

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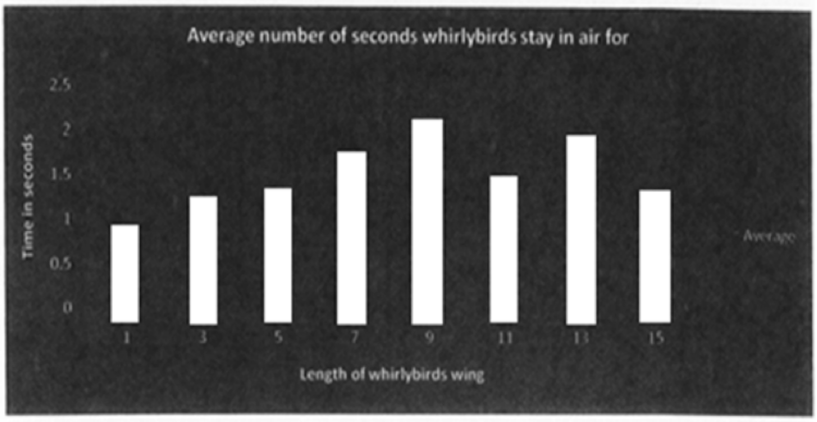
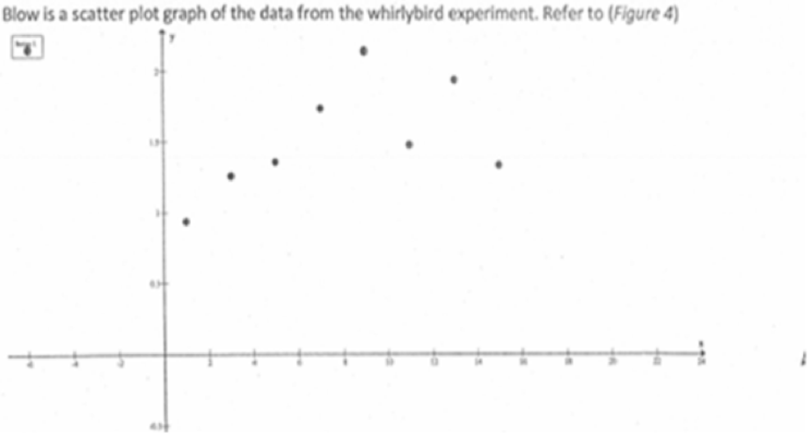
Algebra, measurement, geometry and statistics: Mathematics assignment

It is clear on the graph below that on average with a wing length of 9cm the whirlybird stayed in flight for the longest period of time from the tests conducted. It is also clear that a whirlybird with a length of 1cm on average stayed in flight for the shortest period of time. *(Figure 3)*



Blow is a scatter plot graph of the data from the whirlybird experiment. Refer to *(Figure 4)*



Annotations

Interprets data to draw reasonable conclusions.

Constructs a scatter graph of data (using appropriate technology).

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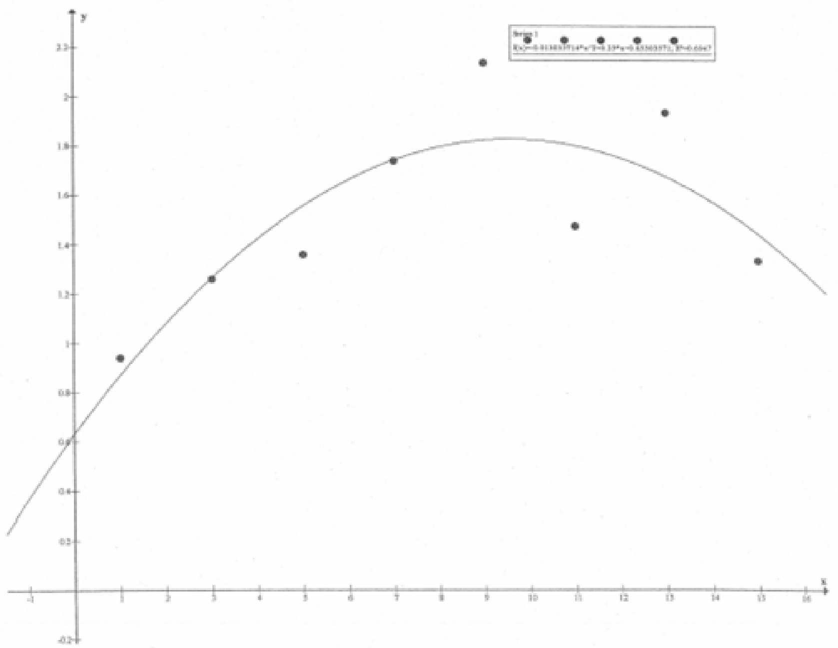
Algebra, measurement, geometry and statistics: Mathematics assignment

Modelling and problem solving

Question 1

Below is a picture of a scatter plot of the averaged data results. *(Figure 5)*



The function of my graph in extended version is: f(x)= - 0.013035714x² + 0.25x + 0.63303571: R2=0.6947

The shortened version of this function is: x= -0 01x² + 0.25x + 0.6

By using the short version of the quadratic function one can determine the turning point of the graph by following the method : x= -b/2a. In my function -b would be -(- and 2a would be .

Therefore $\frac{-0.25}{2* -0.01} = 12.5$

Once one has found the turning point one can determine the length of the whirlybirds length that will stay in flight for the longest period of time. This length would be 12.5cm and by testing my

Annotations

Draws a curve of best fit for data, using appropriate technology.

Finds the equation of parabola, using appropriate technology.

Uses the equation to find the 'x' coordinate of the turning point and so answers the original question about length of whirly bird that gives maximum time in the air.

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Algebra, measurement, geometry and statistics: Mathematics assignment

predictions the whirlybird did in fact stay in the air for the longest period of time on an average of 2.21 seconds.

Part B

a) Can a different number of regions be formed with two tangents? How?

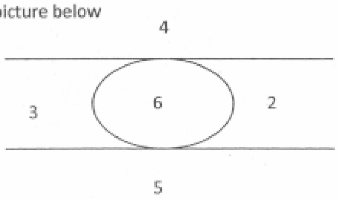
b) What is the least number of regions that can be formed with three tangents?

c) What is the greatest number of regions and the least number of regions that can be produced?

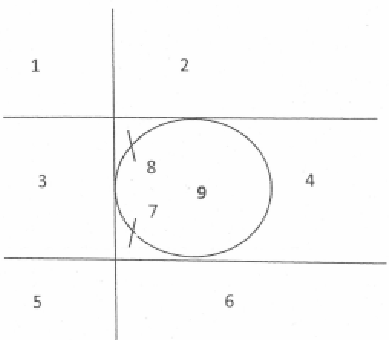
d) Using more tangents, investigate the greatest and least number of regions that can be produced. Display your results in a table.

Knowledge and Procedures

a) Yes a different number of regions can be formed with two tangents. This can be done by putting two parallel lines next to a circle so they do not join, since this would result in more regions. See picture below



b) The least number of tangents that can be formed with three tangents is 9. See picture below.



Annotations

Uses the concept of tangents and regions.

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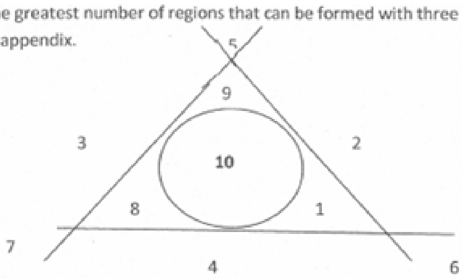
Mathematics

Year 10
Above satisfactory

Algebra, measurement, geometry and statistics: Mathematics assignment

c) The greatest number of regions that can be formed with three tangents is 10. See picture 3 in appendix.



d) Please refer to pictures 4-10 in appendix

(Figure 6)

Tangents	Maximum number of regions
1	3
2	6
3	10
4	15
5	21
6	28

(Figure 7)

Tangents	Minimum number of regions
1	3
2	5
3	9
4	13
5	19
6	25

Modelling & Problem Solving

By testing around with the different tangents and regions ranging up to six (tangents), one could discover that there was a pattern that the regions increased with by with every tangent. There was both a pattern for the maximum and minimum number of regions that could be found for each tangent. When there were 2 tangents the maximum number of regions was 6 and when there were 3 tangents the maximum number was 10. The first step to determining the number of tangents and reasons was to draw diagrams of the circles and tangents and write down the pattern. The next step was to place the maximum and minimum number of regions in a table. With x representing the number of tangents and y representing the maximum number of regions. Please refer to the table below *(Figure 8)*

X (tangents)	1	2	3	4	5	6
Y (max regions)	3	6	10	15	21	28

Annotations

Investigates systematically the relationship between the number of tangents and the number of regions.

Tabulates data.

Describes processes of investigation.

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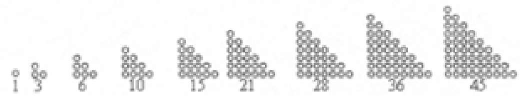
Algebra, measurement, geometry and statistics: Mathematics assignment

One could then search the pattern on Google and determine that the pattern was in fact triangular numbers. This pattern is when the next number increases by one more than the last. For example 6+4=10 and then 10+5=15. The sequence of the triangular numbers comes from the natural numbers, if one always adds the next number, see example below

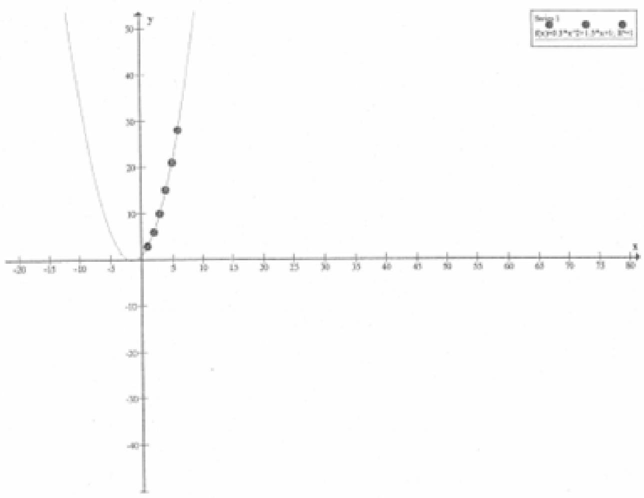
1
1+2=**3**
(1+2)+3=**6**
(1+2+3)+4=**10**
(1+2+3+4)+5=**15**
...

One can illustrate the name triangular number by the following drawing:



The information from the table above (Figure 8) was plotted onto a graph by using a graphing software. X (tangents) and y (maximum number of regions) were used as the variables. After plotting the tangents and regions (1-6) onto the graph the software automatically created a formula that related to the maximum number of regions and the number of tangents. The formula is: **$x = 0.5x^2 + 1.5x + 1$**. With x represents the number of tangents. I changed the formula to represent n (tangents) and thus the formula was created **$n = 0.5n^2 + 1.5n + 1$**. Refer to the graph below *(Figure 9)* to understand the formula and relationship better.

(Figure 9)



Annotations

Links to other contexts by referring to triangular numbers and Pascal's triangle.

Uses functions to predict later results.

Finds a quadratic expression to describe the relationship between the tangents and regions.

Connects data, algebraic functions and graphs.

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Algebra, measurement, geometry and statistics: Mathematics assignment

Using the relationship that was discovered above (**$n = 0.5n^2 + 1.5n + 1$**) one could now determine how many tangents would be needed to form 253 regions. By rearranging my formula one could find that:

Number of regions (R) = $0.5n^2+1.5n+1$ ——— When R=253: $253= 0.5n^2+1.5n+1$

Therefore: $0.5n^2+1.5n-252=0$ ——— (By using the quadratic formula)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

I found that a= 0.5 b= 1.5 c= -252

therefore $x = \frac{-1.5 \pm \sqrt{1.5^2 - 4 \times 0.5 \times -252}}{2 \times 0.5}$

Therefore $x= -1.5 \pm \sqrt{506.25}$ therefore $x= -1.5+ 22.5$ or $x=-1.5- 22.5$

So x= 21 or x = -24 Therefore **x=21** (ignore negative)

As a result 21 tangents formed a maximum of 253 regions. I checked my answer by subbing 21 into the question **$n = 0.5n^2 + 1.5n + 1$:** **$21=0.5(21)^2+ 1.5 (21) + 1$ 21= 253**

The answer was double checked by going through the table below *(Figure 10)* and the answer is correct. Refer to table below.

(Figure 10)

Tangents	Maximum number of regions	Increased by
1	3	
2	6	3
3	10	4
4	15	5
5	21	6
6	28	7
7	36	8
8	45	9
9	55	10

Annotations

Uses knowledge of quadratic functions to solve problems.

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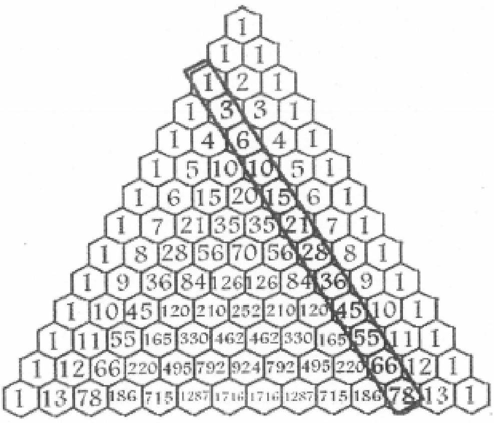
Algebra, measurement, geometry and statistics: Mathematics assignment

10	66	11
11	78	12
12	91	13
13	105	14
14	120	15
15	136	16
16	153	17
17	171	18
18	190	19
19	210	20
20	231	21
21	253	22



(Figure 11)

The numbers in the table are all triangle numbers meaning that they increase by one more than the last.

To determine a relationship which gives the minimum number of regions for an even number of tangents one could place the values in a table and then plot the values on a graph by using a graphing software. See graph below *(Figure 11)*.

Annotations

Investigates and models an authentic situation and formulates a general formula.

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Algebra, measurement, geometry and statistics: Mathematics assignment

Appendix 1:

By checking the answers for the maximum number of regions for tangents ranging from 1 to 6 one must sub the tangent and maximum number of regions into the equation **$n = 0.5n^2 + 1.5n + 1$**. With n= representing the tangents and the number equalling the regions.

For 1 tangents and 3 maximum number of regions:

$n = 0.5n^2 + 1.5n + 1$ $1=0.5(1)^2+1.5(1)+1$ $1=3$ so therefore answer is correct

For 2 tangents and 6 maximum number of regions:

$n = 0.5n^2 + 1.5n + 1$ $2=0.5(2)^2+1.5(2)+1$ $2=6$ so therefore answer is correct

For 3 tangents and 10 maximum number of regions:

$n = 0.5n^2 + 1.5n + 1$ $3=0.5(3)^2+1.5(3)+1$ $3=10$ so therefore answer is correct

For 4 tangents and 15 maximum number of regions:

$n = 0.5n^2 + 1.5n + 1$ $4=0.5(4)^2+1.5(4)+1$ $4=15$ so therefore answer is correct

For 5 tangents and 21 maximum number of regions:

$n = 0.5n^2 + 1.5n + 1$ $5=0.5(5)^2+1.5(5)+1$ $5=21$ so therefore answer is correct

For 6 tangents and 28 maximum number of regions:

$n = 0.5n^2 + 1.5n + 1$ $6=0.5(6)^2+1.5(6)+1$ $6=28$ so therefore answer is correct

By checking the answers for the minimum number of regions for even numbers (2,4,6) one must sub the tangent and minimum number of regions into the equation **$x=0.5x^2+1x+1$**. With x= representing the tangents and the number equalling the regions.

For 2 tangents and 5 minimum number of regions:

$x=0.5x^2+1x+1$ $2=0.5(2)^2+1(2)+1$ $2=5$ so therefore the answer is correct

For 4 tangents and 13 minimum number of regions:

$x=0.5x^2+1x+1$ $4=0.5(4)^2+1(4)+1$ $4=13$ so therefore the answer is correct

For 6 tangents and 25 minimum number of regions:

$x=0.5x^2+1x+1$ $6=0.5(6)^2+1(6)+1$ $6=25$ so therefore the answer is correct

Annotations

Tests formula rigorously.

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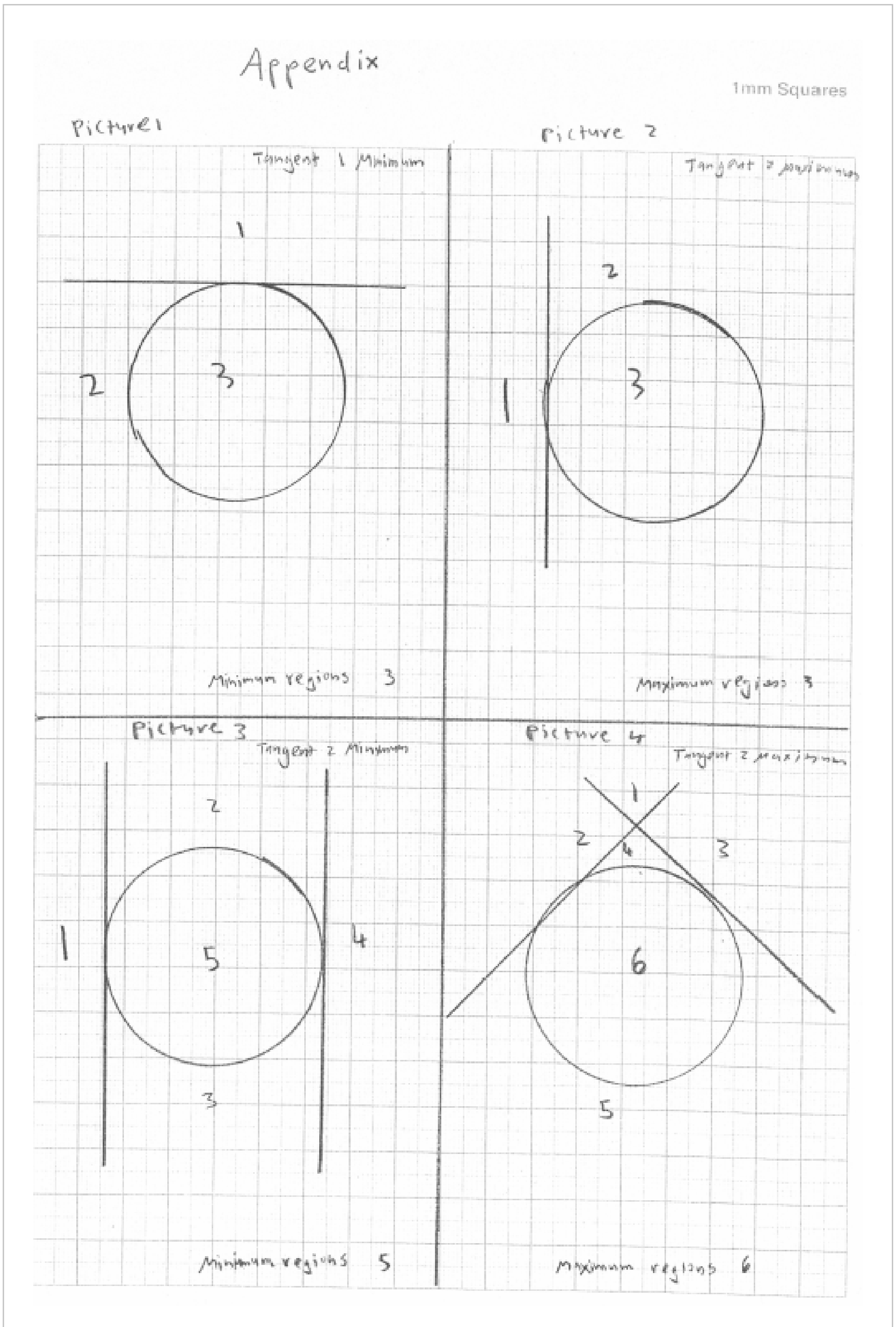
Algebra, measurement, geometry and statistics: Mathematics assignment



Annotations

Provides evidence of investigative process.

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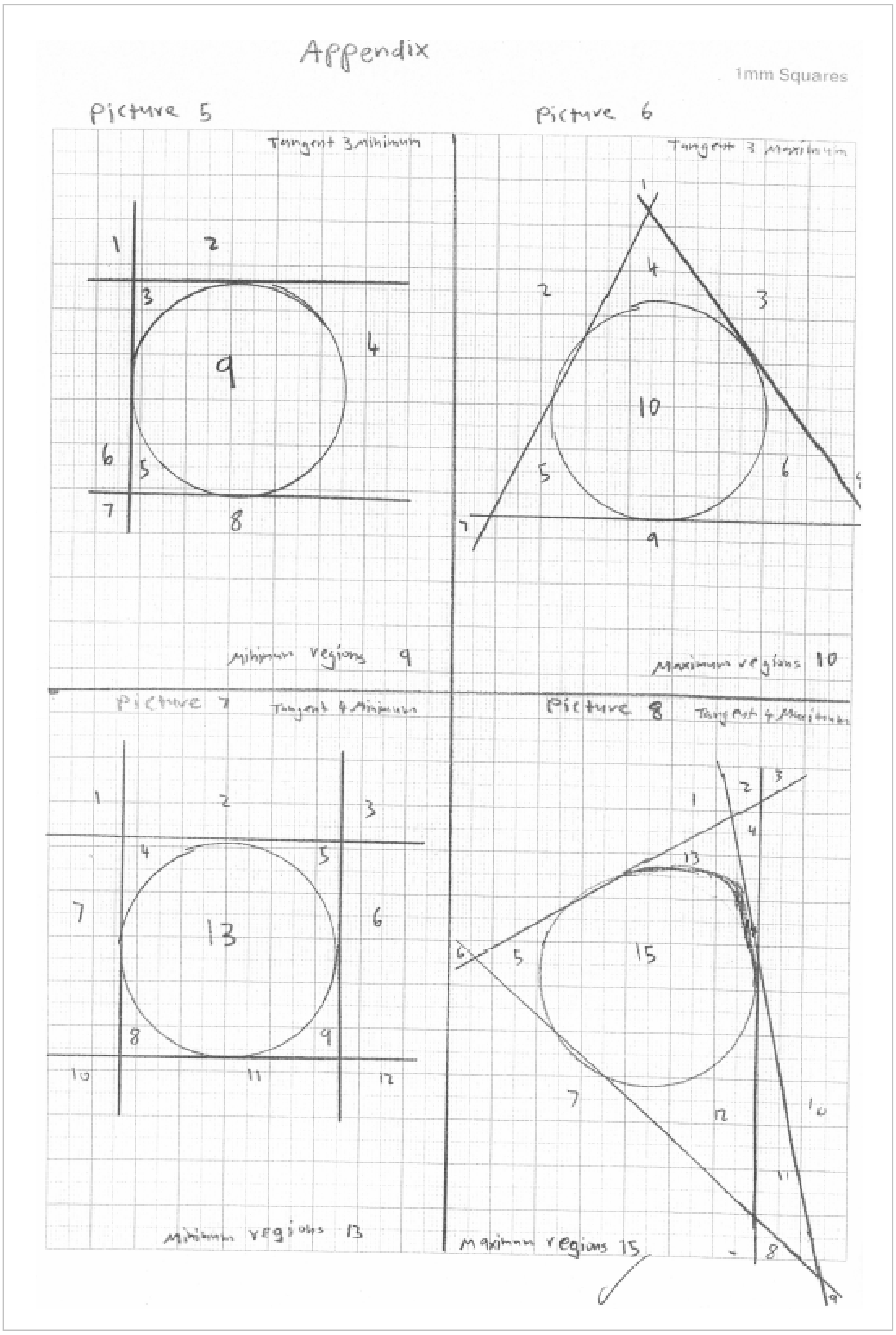
Mathematics

Year 10

Above satisfactory

Algebra, measurement, geometry and statistics: Mathematics assignment



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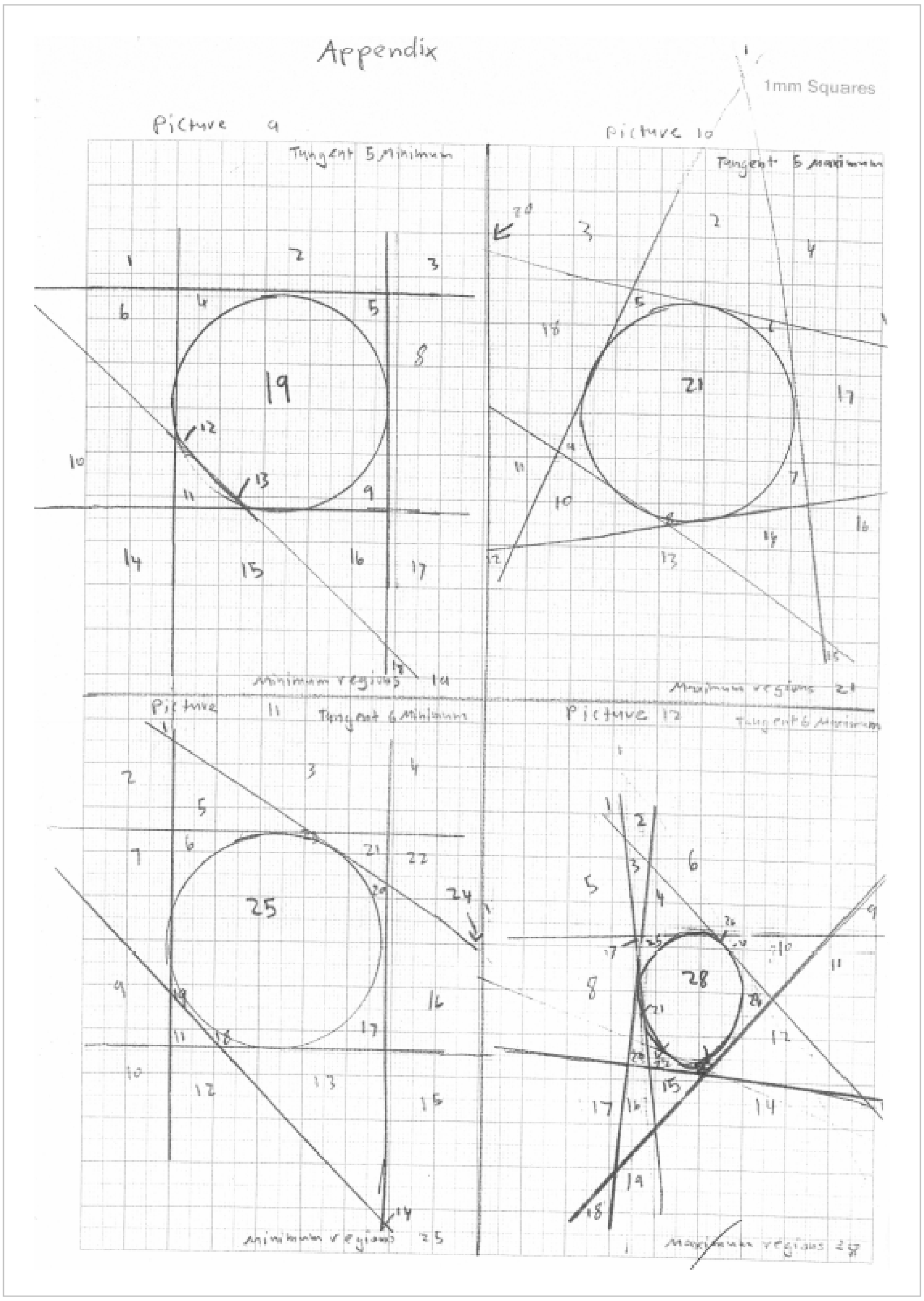

Annotations

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