Mathematics

Year 10

Satisfactory

WORK SAMPLE PORTFOLIO

Annotated work sample portfolios are provided to support implementation of the Foundation – Year 10 Australian Curriculum.

Each portfolio is an example of evidence of student learning in relation to the achievement standard. Three portfolios are available for each achievement standard, illustrating satisfactory, above satisfactory and below satisfactory student achievement. The set of portfolios assists teachers to make on-balance judgements about the quality of their students' achievement.

Each portfolio comprises a collection of students' work drawn from a range of assessment tasks. There is no pre-determined number of student work samples in a portfolio, nor are they sequenced in any particular order. Each work sample in the portfolio may vary in terms of how much student time was involved in undertaking the task or the degree of support provided by the teacher. The portfolios comprise authentic samples of student work and may contain errors such as spelling mistakes and other inaccuracies. Opinions expressed in student work are those of the student.

The portfolios have been selected, annotated and reviewed by classroom teachers and other curriculum experts. The portfolios will be reviewed over time.

ACARA acknowledges the contribution of Australian teachers in the development of these work sample portfolios.

THIS PORTFOLIO: YEAR 10 MATHEMATICS

This portfolio provides the following student work samples:

Sample 1	Algebra: Heptathlon scoring
Sample 2	Statistics: Statistical logic
Sample 3	Probability: Probability and Venn diagrams
Sample 4	Measurement: Trigonometry – why not?
Sample 5	Geometry: Similar or congruent?
Sample 6	Measurement and statistics: How thirsty can you get?
Sample 7	Algebra and geometry: Quadratic equations
Sample 8	Algebra: Simultaneous equations
Sample 9	Geometry: Numerical exercises in geometry
Sample 10	Statistics: Quartiles
Sample 11	Algebra, measurement, geometry and statistics: Mathematics assignment

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This portfolio of student work shows connections between algebraic and graphical representations of relations (WS11). The student solves surface area and volume problems relating to prisms and cylinders (WS6). The student finds unknown values after substitution into formulas (WS1, WS7, WS11), solves pairs of simultaneous equations (WS8) and solves quadratic equations (WS7, WS11). The student applies deductive reasoning to proofs and numerical exercises involving plane shapes (WS9, WS11). The student compares data sets (WS11) and investigates bivariate data where the independent variable is time (WS6). The student describes the relationship between two continuous variables (WS11) and evaluates statistical reports (WS2, WS7). The student calculates quartiles and inter-quartile ranges from a variety of data displays (WS10). The student uses triangle and angle properties to prove congruence and similarity (WS5) and explains how trigonometry can be used to calculate unknown sides and angles in right-angled triangles (WS4). The student lists outcomes for multi-step chance experiments and assigns probabilities for these experiments (WS3).

Algebra: Heptathlon scoring

Year 10 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task/s are highlighted.

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Summary of task

Students had been practising their algebraic skills. They were interested in the results from the athletics carnival and questioned how the heptathlon was scored. They were given this task to complete in class to demonstrate how well they could apply their algebraic skills to a relevant context.

Algebra: Heptathlon scoring

12. Heptathlon Scoring

In the Olympics, "all-round" women athletes compete in the Heptathlon. Held over two days, the athletes compete in the following events:
Day 1— 100 metres hurdles, high jump, shot put, and 200 metres
Day 2— long jump, javelin, and 800 metres.

Points are awarded for each event, and the athlete with the greatest total points is declared the winner. But how do they decide how many points to award to a particular performance, and how can you compare events? For example, how do you compare a 25 second performance in the 200 metre event with throwing the discus 75 metres?

Mathematics, Physics, and Computer Modelling were used by Dr Karl Ulbrich to create the current scoring system, which attempts to make fair comparisons between events.

There are three main rules used for calculating points in the seven events:

- The track events (200 m; 800 m; and 100 m hurdles): $P = a \times (b - T)^c$
- The jump events (high jump and long jump): $P = a \times (M - b)^c$
- The throwing events (shot put and javelin): $P = a \times (D - b)^c$

In these rules, P is the point score; T is the time in seconds; M is measurement in cm; and D is the distance in metres. a, b, and c are different for each event, as shown in the table:

EVENT	a	b	c
200 m	4.99	42.5	1.81
800 m	0.11	254	1.88
100 m hurdles	9.23	26.7	1.84
high jump	1.85	75.0	1.35
long jump	0.19	210	1.41
shot put	56.0	1.50	1.05
javelin	16.0	3.80	1.04

For example, a 29.30 second performance in the 200 metre event, would give the following number of points:

$$P = 4.99 \times (42.5 - 29.3)^{1.81}$$
$$= 532.6$$

Annotations

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Algebra: Heptathlon scoring

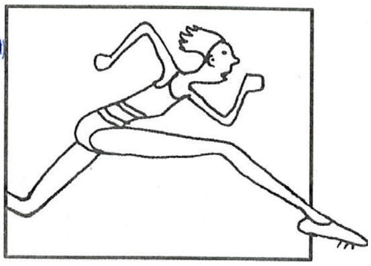
1. Write the appropriate rule to work out the points in each of the following cases, and then work out the points, using your calculator:

- a 32-second performance in the 200 m

$P = a \times (b - T)^c$

$= 4.99 \times (42.5 - 32)^{1.81}$

$= 351.927477742$

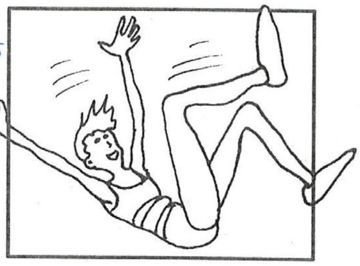


- a high jump of 1.80 m

$P = a \times (M - b)^c$

$= \overset{1.85}{a \times} (1.80 - 75)^{1.35}$

$= -608.50154861\overline{5}$

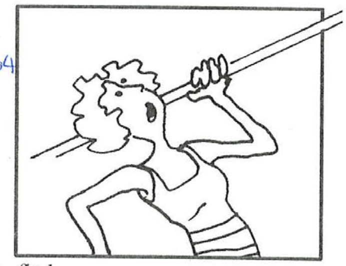


- a javelin throw of 65 m

$P = a \times (D - b)^c$

$= 16.0 \times (65 - 3.80)^{1.04}$

$= 1154.36051082$



2. Using whatever method you think appropriate, find

- the time a runner would need to score 1000 points in the 200 m.

$1000 = a \times (b - T)^c$

$1000 = 4.99 \times (42.5 - T)^{1.81}$

$(42.5 - T)^{1.81} = \frac{1000}{4.99}$

$(42.5 - T)^{1.81} = 200.400801603$

using guessing and my calculator

$T \doteqdot 23$ seconds

continued ...

Annotations

Calculates the points.

Completes a calculation correctly but with incorrect substitution by not converting metres to centimetres.

Calculates answer correctly.

Uses correct reasoning to come up with an approximate answer.

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Algebra: Heptathlon scoring

- the distance in the long jump which would score the same number of points as a time of 1:55 in the 800 metre event.

Long jump

$P = a \times (M - b)^c$

$1881 = 0.19 \times (M - 210)^{1.05}$

800 m

$P = a \times (b - T)^c$

$= 0.1k(254 - 1.55)^{1.88}$

$= 0.11 \times (254 - 75.5)^{1.88}$

$= 1881.3627$

3. At the 1988 Seoul Olympics, just before the last event (the 800 m), Jackie Joyner-Kersee of the USA needed 894 points to break the world record.

- What was the slowest time which Jackie could run and still break the world record

$894 = a(b - T)^c$

$894 = 0.11\,(254 - T)^{1.88}$

$(254 - T)^{1.88} = 8127.272727$

guessing and checking on my calculator

$T \doteqdot 130$ secs.

$=$ 2 mins 10 secs.

[Incidentally, Jackie ran 2:08.51, giving her a final total of 7291 points—still the current world record]

4. Now make up your own challenging question about the Heptathlon which can be answered using the information you have been given about the scoring system, and provide the calculations and the answer.

How many points would a throw of 52 m be in Javelin?

$P = a \times (D - b)^c$

$P = 16.0 \times (52 - 3.8)^{1.04}$

$P = 16.0$

$= 900.510751604$

Annotations

Attempts to reason through this problem but does not complete the answer despite having correct working.

Calculates an approximate answer and converts answer to minutes and seconds.

Develops a simple problem and answers correctly.

Statistics: Statistical logic

Year 10 Mathematics achievement standard

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Summary of task

Students had spent some time looking at media reports of statistical data. This task was given as a 10-minute test to evaluate how students could discern the facts from some statements.

Statistics: Statistical logic

Statistical Logic

Please comment on the three following uses of statistics. Does the logic in the statement make sense? Please explain your reasoning.

- "Young people account for 30% of all road accidents. Of course, this means that older drivers account for 70% of all road accidents- many more. The older drivers should get off the road and leave it to us young ones!"

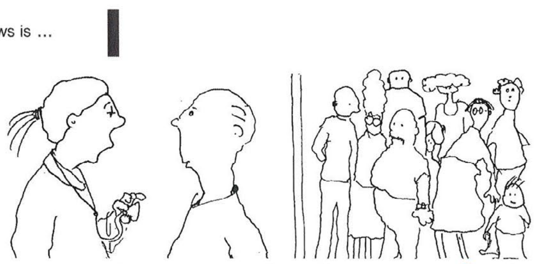
This makes an erroneous assumption that if older drivers were not driving there would be fewer accidents and ignores ~~that~~ the fact that there are many categories of drivers, not just categories based on age.

- "What is happening to our school system? It's a disgrace-50% of our students are below the school average!"

given that the measure of 50% provides a centre for the data the statement is illogical.

- A doctor informed a patient that he had a life-threatening disease, for which about 9 out of 10 patients usually died. "The good news", the doctor said, "is that my last nine patients have all died!"

The good news is ...



given there is a 90% chance of death there is a good possibility that this patient will also die.

Adapted from and used with kind permission Curriculum K – 12 Directorate NSW Department of Education and Training

Annotations

Makes a logically valid first point in the argument.

Indicates understanding of the average as a measure of the centre of the data and that this is the basis of the error in the quote.

Most of the points in the student counterargument are correct and the last point in particular addresses the issue at the heart of the error in the quote.

Probability: Probability and Venn diagrams

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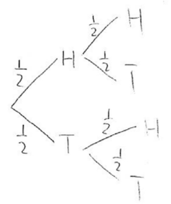
Summary of task

Students had completed a unit of work on probability. They had spent several lesson applying their knowledge to experiments, recording results and calculating probabilities. Students were encouraged to reason through some problems and justify their conclusions using mathematical language. This task was given as a test during class time.

Probability: Probability and Venn diagrams

Knowledge and Understanding
Question 1

a) Use a tree diagram to show the sample space for tossing 2 coins **simultaneously**.



b) Determine the probability of obtaining 1 head and a tail

$P(1H \text{ and } 1T) = P(1H) + P(\cancel{B}1T)$

$= P(H) + P(T)$

$= \frac{1}{2} \times \frac{1}{2} + \cancel{\frac{1}{2}\times\frac{1}{2}} \times 2$

$= \frac{1}{2}$

c) Determine the probability of obtaining at least 1 head.

$\cancel{P(\text{at least } 1H) = (P(HH) + P(HT)) \times 2}$

$\cancel{= (0.25 + 0.25) \times 2}$

$\cancel{= 1}$

$P(\text{at least one } H) = P(HH) + P(HT) \times 2$

$= 0.25 + 0.25 \times 2$

$= 0.75$ or $\frac{3}{4}$ (1½)

Question 2

An equal- sector spinner containing numbers 1, 2, 3, and 4 is spun and an unbiased coin is tossed.

a) Draw a two way table to represent the sample space.

	1	2	3	4
H	H,1	H,2	H,3	H,4
T	T,1	T,2	T,3	T,4

b) Determine the probability of obtaining a head and a 4

$P(H,4) = \frac{1}{8}$

c) Determine the probability of obtaining a tail or an even number

$P(T, \text{ or even } \#) = P(T) + P(\text{even} \#)$

$= \frac{4}{8} + \frac{4}{8}$

$= 1$

Annotations

Constructs probability tree diagram.

Constructs and completes table correctly to represent sample space.

Interprets table correctly to calculate probability of event described.

Recognises that there are two ways of obtaining one head and one tail and calculates the answer.

Calculates the probabilities of each event correctly but assumes the events described are mutually exclusive.

Demonstrates understanding of the term 'at least' and calculates the probability.

Probability: Probability and Venn diagrams

Question 3

A box contains 3 red, 2 blue and 1 yellow marble. Two marbles are drawn **simultaneously** from the box. Determine the probability that they will be:

a) both red

RRR BB
Y

$P(R \text{ and } R) = P(R) \times P(R)$

$= \frac{3}{6} \times \frac{3}{6}$

$= 0.25$ or $\frac{1}{4}$

b) red and blue

$P(R \text{ and } B) = P(R) + P(B)$

$= \frac{3}{6} \times \frac{2}{6}$

$= \frac{1}{6}$

c) both the same colour

$P(\text{same colour}) = P(RR) + P(BB)$

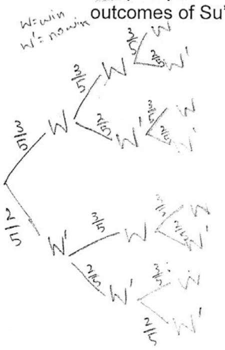
$= \frac{1}{4} + \frac{1}{6}$

$= \frac{5}{12}$

Question 4

In a chess tournament each contestant must play three others. Su estimates that she has a 60% chance of winning a game.

a) Draw a tree diagram and list the sample space for the possible outcomes of Su's three games.



$60\% = 0.6$
$= \frac{3}{5}$

Sample space:
W, W, W
W, W, W'
W, W', W
W, W', W'
W', W, W
W', W, W'
W', W', W
W, W', W'

b) Find the probability of Su winning at least 2 games.

$P(\text{win at least 2 games}) =$

$P(W, W, W) + P(W, W, W') \times 2$

$= \left(\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}\right) + \left(\frac{3}{5} \times \frac{3}{5} \times \frac{2}{5}\right) \times 2$

$= 0.216 + 0.144 \times 2$

or $\frac{27}{125} + \frac{18}{125} \times 2$

$= 0.504$ or $\frac{63}{125}$

Annotations

Constructs and interprets tree diagram for three-stage experiment.

Multiplies probabilities of each event but the draw is done without replacement so calculation should be 3/6 x 2/5.

Realises that two red marbles or two blue marbles are the two possible solution outcomes.

Demonstrates understanding of the words 'at least'. Calculates some probabilities from the tree diagrams.

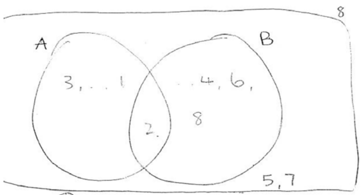
Probability: Probability and Venn diagrams

Question 5

An eight-sided die is rolled with faces numbered 1 – 8.

A is the event 'numbers less than 4' and B is the event 'numbers that are multiples of 2'.

a) Draw a Venn diagram to represent the above information.



b) Calculate the probability that the number is less than 4 given that it is a multiple of 2.

P(<4 given multiple of 2)

$= \frac{P(\text{less than } 4)}{P(\text{multiple of } 2)}$

$= \frac{1}{4}$

c) Determine that probability that the number is a multiple of 2 given it is less than 4.

P(multiple of 2 given less than 4)

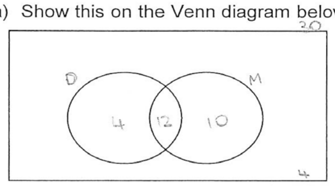
$= \frac{P(\text{multiple of } 2)}{P(\text{less than } 4)}$

$= \frac{1}{3}$

Question 6

A teacher surveys her class of students about their chocolate preferences. Out of 30 students, 16 students liked dark chocolate (D) and 22 students liked milk chocolate (M). Only 4 of the students surveyed liked neither milk nor dark chocolate.

(a) Show this on the Venn diagram below.



30 students

22 M
16 D
4 - none

30 - 4 = 26
∴ only 26 like chocolate
22 + 16 = 38
38 - 26 = 12 - in middle
22 - 12 = 10 M
16 - 12 = 4

(b) Determine the probability that a randomly selected student from the class likes:

(i) both milk and dark chocolate

P(M and D) = P(M) + P(D)

$= \frac{12}{30}$

$= \frac{2}{5}$

(ii) milk chocolate only

$P(M) = \frac{10}{30}$

$= \frac{1}{3}$

Annotations

Uses Venn diagram to calculate a solution.

Demonstrates use of diagram to calculate the solution.

Measurement: Trigonometry – why not?

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Summary of task

Students had been investigating trigonometry. They had looked at the applications and use of trigonometry. Students were asked to complete these questions as a formative assessment task to give the teacher an indication of how much revision the student required.

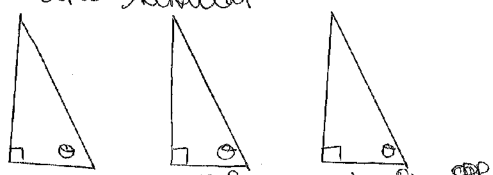
Measurement: Trigonometry – why not?

Trigonometry- Why not?

A person was quoted in the local paper as saying 'the things you learn at school are just not relevant when you leave school'. My Maths teacher was just horrified and asked the following questions:

1. Write down all that you know about the trigonometric ratios.
2. Why would people need these ratios in life outside school?

1. There are 3 trigonometric ratios. They relate all the sides and angles of a right angled triangle together. This is because the triangles are similar



$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

You can use trigonometry to calculate the angle or side of any triangle. This is because the size of the angles always have to add up to 180° and the side are in the same ratio.

2. Trigonometry is used in all construction, transport and travel.

Annotations

Explains the three trigonometric ratios using a diagram.

Recognises the relationship between sum of angles of triangles and the similarity of triangles.

Identifies the use of trigonometry in various relevant contexts but does not expand on the usages.

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Geometry: Similar or congruent?

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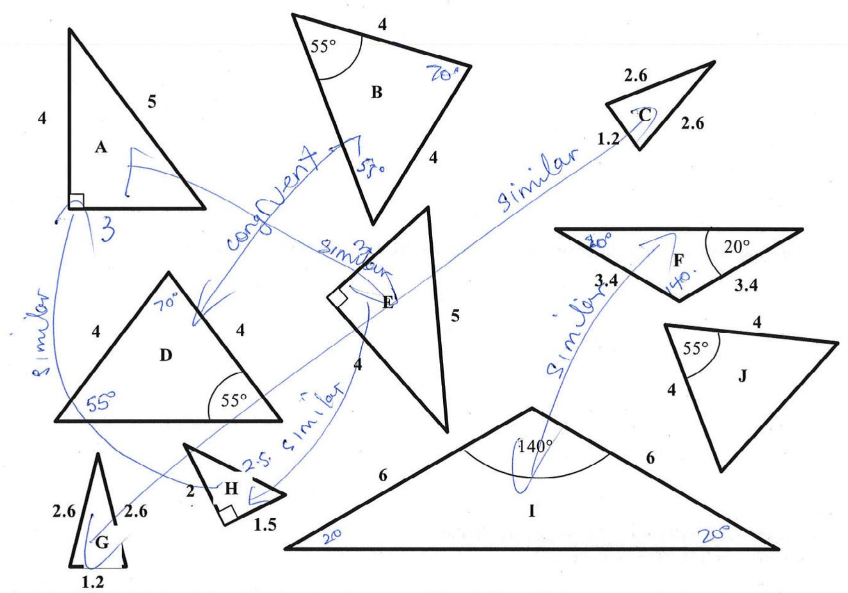
Summary of task

Students had studied both similarity and congruence during the year. They were asked to make connections between the two concepts and to complete a task which involved both concepts. The teacher wanted to ensure that students could clearly identify the difference between similarity and congruence.

Geometry: Similar or congruent?

Similar? Congruent?

1. Consider the following triangles and complete the table below. (All lengths are in centimetres)



Which triangles are similar?	Which triangles are congruent?	Reasons for congruency
A and H A and E +.:. H C and G I and F	B and D	SAS test.

Annotations

Identifies which triangles are similar and which are congruent.

Uses the correct test for congruence but does not explain how this test was used.

Identifies similar and congruent triangles from the diagram above.

Measurement and statistics: How thirsty can you get?

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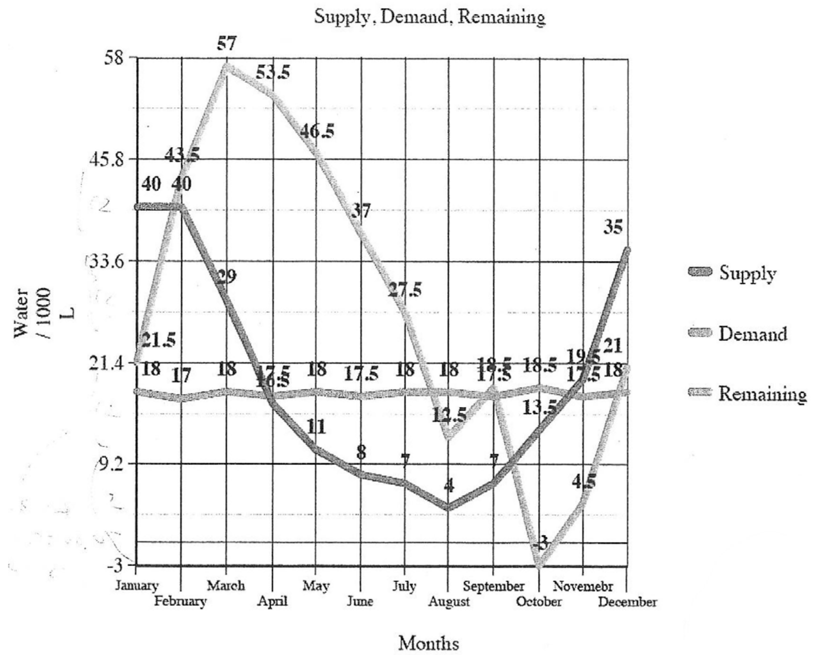
Summary of task

Students had spent two weeks investigating surface area and volume. They were given this task as an assignment to apply the skills they had learnt in class to a real-world problem. They were asked to solve the problem using their knowledge of surface area and volume to perform calculations and justify their results.

Measurement and statistics: How thirsty can you get?

HOW THIRSTY WILL YOU GET?

Month	Monthly water supply	Volume of rain each month	Water consumed	Water left
January	200.013mm	40002.6 L	18060.6 L	21942 L
February	200.013mm	40002.6 L	16895.4 L	43884 L
March	144.837mm	28967.4 L	18060.6 L	54790.8 L
April	82.764mm	16552.8 L	17478 L	53865.6 L
May	55.176mm	11035.2 L	18060.6 L	46840.2 L
June	41.382mm	8276.4 L	17478 L	37638.6 L
July	34.485mm	6897 L	18060.6 L	26475 L
August	20.691mm	4138.2 L	18060.6 L	12552.6 L
September	34.485mm	6897 L	17478 L	1971.6 L
October	68.97mm	13794 L	18060.6 L	-2295 L Buy 5KL extra water Becomes 2705L
November	96.558mm	19311.6 L	17478 L	4538.6 L
December	172.425mm	34485 L	18060.6 L	20963 L



Annotations

Shows values with appropriate units in tabular form that demonstrate understanding of the problem but with a few incorrect values.

Uses technology to graph water supply, water demand and water remaining against the independent variable, time, on the same axes but with some values inconsistent with the table.

Measurement and statistics: How thirsty can you get?

The tank I have chosen to buy is 60 KL at a cost of $1800. I assume that the rainfall and the daily consumption would be consistent. The rainfall I have been given is for this year but if you were choosing a tank, you would probably have to look at the pervious 5 -year rainfall patterns to compare.

60 KL tank = $1800 + $50 for 5 KL of extra water
$1800 + 50 = $1850
Therefore the total cost for the water tank and the extra water will be $1850.

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January	200.013mm	40002.6 L	18060.6 L	21942 L
February	200.013mm	40002.6 L	16895.4 L	45049.2 L
March	144.837mm	28967.4 L	18060.6 L	55956 L
April	82.764mm	16552.8 L	17478 L	55030.8 L
May	55.176mm	11035.2 L	18060.6 L	48005.4 L
June	41.382mm	8276.4 L	17478 L	38803.8 L
July	34.485mm	6897 L	18060.6 L	27640.2 L
August	20.691mm	4138.2 L	18060.6 L	13717.8 L
September	34.485mm	6897 L	17478 L	3136.8 L
October	68.97mm	13794 L	18060.6 L	-1129.8 L Buy 5KL extra water Becomes 3870.2 L
November	96.558mm	19311.6 L	17478 L	5703.8 L
December	172.425mm	34485 L	18060.6 L	22128.2 L

The new cost of using this tank in the second year of operation is $50 for purchasing extra water

Annotations

Identifies some assumptions used in suggesting an appropriate-sized tank.

Identifies a more appropriate method that could be used to answer the question.

Correctly calculates the cost of the suggested tank in the first year using the table of water supply/water consumption.

Applies a correct approach to calculate each variable throughout the second year, but does not take into account the amount of water remaining in the tank at the end of the first year.

Measurement and statistics: How thirsty can you get?

Two other tank choices that could be purchased are a 40 KL tank, costing $1300 or a 100 Kl tank, costing $2700.
Although the 40 KL tank is much cheaper then the 60 KL tank, it would not be ideal for the amount of rainfall. The tank would over flow in February, March, April and May. This would mean that the water would be wasted.
The 100 KL tank would also not be ideal for this family as the tank would be too big, wasting space and costing extra money that does not need to be spent.

"Tanks 4 your loot"

64 m^2 tank = $1200

$V = \pi r^2 \times h$

$V = \pi r^2 \times 5$

$60 = 3.1416 \times r^2 \times 5$

$60 \div 15.708 = r^2$

$= 3.819$

$\sqrt{3.819}$

$= 1.954$ m

$r = 1.954$ / $D = 3.91$ m

Therefore using a tank of 5 m high and a radius of 1.954/diameter of 3.91 m then the TSA is 3.1416 × 3.91 × 5

TSA = 61.4 m^2

Annotations

Communicates reasons why two alternative tank sizes are less suitable than the chosen size.

Attempts to calculate the surface area of a cylindrical tank using its volume and the assumption that its height is 5 metres, but uses an incorrect formula for the surface area.

Measurement and statistics: How thirsty can you get?

"Tanks R us"
60 KL tank = \$1800
Therefore it is cheaper to buy a tank from "tanks 4 your loot".
Tanks 4 your loot 64 m tank = \$1200 and Tanks R us 60 KL tank = \$1800.

Compare tank shapes and determine which shape gives you more volume for least surface area.

Cylinder tank:

$V = \pi r^2 h$
$V = 3.1416 \times 4 \times 5$
$V = 62.83$ KL
$TSA = 2\pi r^2 + 2\pi r^2 h$
$TSA = 25.13 = 62.83$
$= 87.96\ m^2$

Rectangular prism:

$V = L \times W \times H$
$V = 3 \times 4 \times 5$
$V = 60$ KL
$TSA = 2 (L \times W) + 2 (L \times H) + 2 (W \times H)$
$TSA = 2 \times 5 \times 3 + 2 \times 5 \times 4 + 2 \times 3 \times 4$
$TSA = 30 + 40 + 24$
$TSA = 94\ m^2$

Therefore my conclusion is that based on least surface area a cylinder tank holds more volume and is more efficient and economical then a rectangular prism tank.

Annotations

Compares the surface areas of a cylinder and a rectangular prism of approximately equal volume by assuming particular values for the dimensions of the solids, but does not consider other values for the volume or other tank shapes.

Draws a correct conclusion from the calculations shown.

Algebra and geometry: Quadratic equations

Year 10 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task/s are highlighted.

By the end of Year 10, students recognise the connection between simple and compound interest. They solve problems involving linear equations and inequalities. They make the connections between algebraic and graphical representations of relations. Students solve surface area and volume problems relating to composite solids. They recognise the relationships between parallel and perpendicular lines. Students apply deductive reasoning to proofs and numerical exercises involving plane shapes. They compare data sets by referring to the shapes of the various data displays. They describe bivariate data where the independent variable is time. Students describe statistical relationships between two continuous variables. They evaluate statistical reports.

Students expand binomial expressions and factorise monic quadratic expressions. They find unknown values after substitution into formulas. They perform the four operations with simple algebraic fractions. Students solve simple quadratic equations and pairs of simultaneous equations. They use triangle and angle properties to prove congruence and similarity. Students use trigonometry to calculate unknown angles in right-angled triangles. Students list outcomes for multi-step chance experiments and assign probabilities for these experiments. They calculate quartiles and inter-quartile ranges.

Summary of task

Students had spent some time solving quadratic equations and solving problems that required them to form a quadratic equation as a way to find a solution to a problem. This task was set as a class test that took 20 minutes.

Mathematics

Year 10
Satisfactory

Algebra and geometry: Quadratic equations

1 Which of the options given are the solution(s) to each of these equations?
*(Circle **all** that apply.)*

$3y + 2y = 92 - 7$
$5y = 85$
$y = 17$

(a) $3y + 7 = 92 - 2y$

A 6 B 7 C −6 D −7 E none of the above

(b) $x^2 - 24 = 5x$

A 8 B 12 C −2 D −3 E none of the above

(c) $m^2 = -100$

A 5 B −10 C 10 D −50 E none of the above

(d) $x^3 - 2x^2 - 11x + 12 = 0$

A −4 B 4 C −3 D 1 E none of the above

2 Provide **exact** solutions (i.e. $\sqrt{5}$, not 2.236) to the following equations.

a $y^2 = 4$

$y = \pm 2$

b $x^2 - 21 = 0$

$x^2 = 21$

$x = \pm\sqrt{21}$

c $\dfrac{2x^2 + 7}{3} = 100$

$2x^2 + 7 = 100 \times 3$

$2x^2 = 293$

$x^2 = \frac{293}{2} = 146.5$

~~$\pm\sqrt{146.5}$~~

$x = \pm\sqrt{\frac{(100 \times 3) - 7}{2}}$

d $(a + 4)(a - 1) = 0$

~~$(a^2 - a + 4a - 4$~~

$= a^2 + 3a - 4$

$\therefore\ a + 4 = 0$ or $a - 1 = 0$

$a = -4$ or $a = 1$

e $6(2m - 1)(3m + 4) = 0$

$2m - 1 = 6$ or $3m + 4 = 6$

$2m = 7$ or $3m = 2$

$m = \frac{7}{2}$ or $m = \frac{2}{3}$

Annotations

Uses an efficient method to solve a simple linear equation correctly.

Demonstrates an understanding that quadratic equations can have two solutions and correctly determines the two solutions.

Recognises that some quadratic equations cannot be solved for real solutions.

Solves simple quadratic equations, demonstrating understanding of the concept of an exact solution.

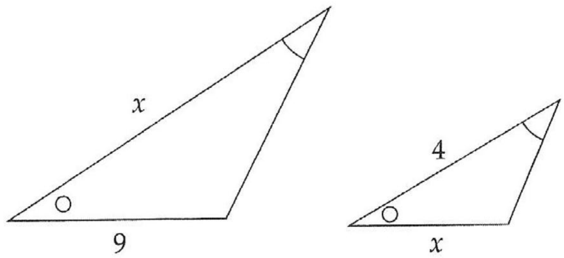
Demonstrates a correct procedure for solving a simple quadratic equation involving a fraction, but does not leave the answer in simplest exact form.

Demonstrates some knowledge of how to solve quadratic equations given in factored form, but is unable to apply this knowledge successfully to more complex equations given in factored form.

Algebra and geometry: Quadratic equations

3 In the following diagram, the two triangles are similar.

Write an appropriate equation and solve it to find the value of x.



$$\frac{x}{9} = \frac{4}{x}$$

$$x^2 = 36$$

$$x = \sqrt{36}$$

$$\therefore x = 6$$

Annotations

Applies knowledge of similar figures to form a quadratic equation.

Indicates only solution to the quadratic equation used to solve the problem.

Obtains the correct solution for the context but does not justify choice of the positive solution.

Algebra: Simultaneous equations

Year 10 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task/s are highlighted.

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>
> *Students expand binomial expressions and factorise monic quadratic expressions. They find unknown values after substitution into formulas. They perform the four operations with simple algebraic fractions. Students solve simple quadratic equations and pairs of simultaneous equations. They use triangle and angle properties to prove congruence and similarity. Students use trigonometry to calculate unknown angles in right-angled triangles. Students list outcomes for multi-step chance experiments and assign probabilities for these experiments. They calculate quartiles and inter-quartile ranges.*

Summary of task

Students completed a unit of work on equations. The unit included looking at different methods of solving linear and simultaneous equations, including applying these techniques to solve word problems. The students were given 20 minutes to complete this assessment task.

Algebra: Simultaneous equations

1 How many solutions does the equation $7x + 5y = 24$ have? Explain.

$7x + 5y - 24 = 0$

$= \dfrac{-5 \pm \sqrt{25 - (-24 \times 7 \times 4)}}{2 \times 7}$

$x = \dfrac{-5 \pm \sqrt{697}}{14}$ ∴ 2 solutions.

2 Solve $2x - y = 5$ if:

(i) $x = 5$

$2 \times 5 - y = 5$

$-y = 5 - 10$

$y = 5$

(ii) $y = -2$

$2x - (-2) = 5$

$x = \dfrac{5 - 2}{2}$

$x = \dfrac{3}{2}$

3 Solve the following equations simultaneously:

(i) $3x + y = 10$ and $x - y = -2$

$3x + y = 10$ ①

$x = -2 + y$ ②

sub ② into ①

$3(-2 + y) + y = 10$

$-6 + 3y + y = 10$

$y = \dfrac{10 + 6}{4}$

$y = 4$

sub $y = 4$ into $x - y = -2$

$x = -2 + 4$

$x = 2$

(ii) $2x + 9y = 43$ ① and $y = x - 1$ ②

sub ② into ①

$2x + 9(x - 1) = 43$

$2x + 9x - 9 = 43$

$x = \dfrac{52}{11}$

sub $x = \dfrac{52}{11}$ into ②

$y = \dfrac{52}{11} - 1$

$y = 3\dfrac{8}{11}$

Annotations

Attempts to answer the question but chooses an incorrect approach.

Finds the value of one variable given the value of the other.

Uses the substitution method to solve the pairs of simultaneous equations.

Algebra: Simultaneous equations

$x = \frac{31-5y}{3}$ ②

(continued)

(iii) $7x - 3y = -20$ ① and $3x + 5y = 31$ ②

$7x - 3y = -20$ ①

$x = \frac{31-5y}{3}$ ②

sub ② into ①

$7\left(\frac{31-5y}{3}\right) - 3y = -20$

$\frac{217 - 35y - 9y}{3} = -20$

$-\frac{44y}{3} = -20 - \frac{217}{3}$

$y = -92\frac{1}{3} \times \frac{3}{-44}$

$y = 6\frac{13}{44}$

sub $y = 6\frac{13}{44}$ into $7x - 3y = -20$

$7x - 3\left(6\frac{13}{44}\right) = -20$

$x = \frac{-20 + 18\frac{39}{44}}{7}$

$x = -\frac{7}{44}$

Choose **one** of the problems below and use simultaneous equations to solve it.

I A piggybank contains only 20c and 50c coins. There are 60 coins in total and their overall value is $18.60. Determine the contents of the piggybank.

II A man is currently six times older than his granddaughter. Thirteen years ago, he was 18 times older than her. What are their current ages?

let number of 20c be x

number of 50c be $60 - x$

$x + (60 - x) = 1860$

~~$x + (60 - x) = 60$~~

no. of 20c = 38

no. of 50c = 22

Annotations

Demonstrates sound algebraic skills to accurately solve a pair of simultaneous equations.

Interprets a word problem and establishes an efficient procedure to find a solution but is unable to proceed and appears to find the answer by guess and check.

Geometry: Numerical exercises in geometry

Year 10 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task/s are highlighted.

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Summary of task

Students had studied a unit of work on geometrical reasoning. An assessment task was given at the end of the unit. Students were expected to spend between 10 and 15 minutes to complete this task.

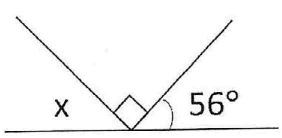
Mathematics

Year 10
Satisfactory

Geometry: Numerical exercises in geometry

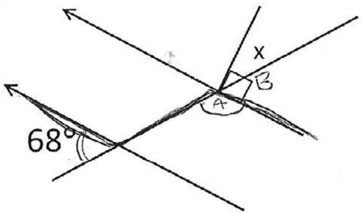
Calculate the values of the unknown angles x in each of the diagrams below.

a)



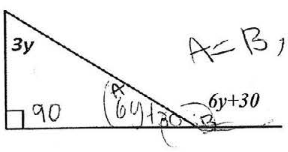
$x + 90 + 56 = 180$
$x + 146 = 180$
$x = 180 - 146$
$x = 34°$

b)



$A = 112°$
$B = 68°$
$\therefore x = 90 - 68$
$x = 22°$

Calculate the value of *y* in the following diagram.



$A = B,$

$3y + 6y + 30 + 90 = 180$
$9y + 120 = 180$
$9y = 180 - 120$
$\frac{9y}{9} = \frac{60}{9}$
$y = 7°$

Annotations

Recognises the straight angle and establishes an equation to solve the problem.

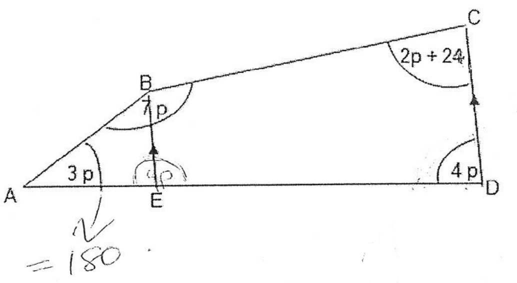
Uses a sequence of angle properties to obtain the correct value.

Recognises that the angle sum of a triangle can be used but incorrectly states that the interior angle of the triangle is equal to the exterior angle.

Geometry: Numerical exercises in geometry

(a) *Use algebraic methods to find the value of p.*

(b) *Determine the size of ∠ABE.*



A) $3p + 7p + 2p + 4p + 24 = 360$

$16p + 24 = 360$

$16p = 360 - 24$

$\frac{16p}{16} = \frac{336}{16}$

$p = 21$

B) ~~3p + 3.5p + x = 180~~

~~6.5p + x = 180~~

~~6.5 × 21 + x = 180~~

~~136.5~~

Annotations

Applies the angle sum of a quadrilateral to establish an equation and solve the problem.

Attempts to solve the problem but is unable to apply the angle properties required.

Mathematics

Year 10
Satisfactory

Statistics: Quartiles

Year 10 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task/s are highlighted.

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Summary of task

Students had spent some time studying statistics, including the calculation of quartiles and inter-quartile ranges in five-number summaries from a variety of data displays. This task was set for students to complete in 20 minutes of class time.

Mathematics

Year 10
Satisfactory

Statistics: Quartiles

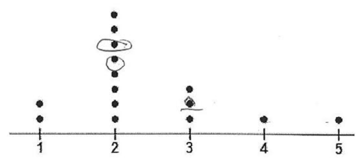
In this work sample, you will calculate the five-number summary for several data sets. The data sets are labelled **A–G**, and your answers are to be placed in the table in the middle of the page.

A	4	7	8	8	10	16	18	19	19	20	23	23	24			
B	4	7	8	8	10	16	18	19	19	20	23	23	24	27		
C	4	7	8	8	10	16	18	19	19	20	23	23	24	27	28	
D	4	7	8	8	10	16	18	19	19	20	23	23	24	27	28	29

E

Score	Frequency	
1	8	
2	11	19
3	15	24
4	2	26
5	9	35
6	15	50
7	18	68

F

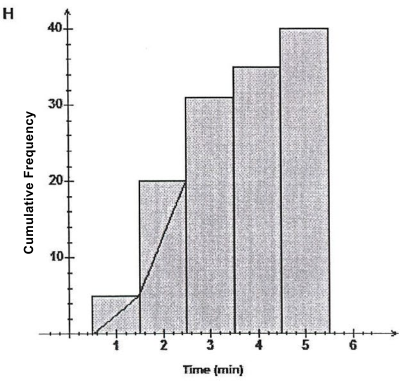


G

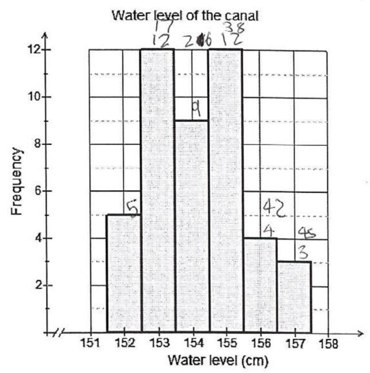
Stem	Leaf
0	3 4
0	8 9 9
1	2 2 3 4 4 4
1	5 6 7 7 7 8 8
2	0 1 1 1 2 2 3 4

	Min	Q1	Med	Q3	Max	IQR
A	4	8	18	20	24	12
B	4	8	18.5	23	27	15
C	4	8	19	23	28	15
D	4	9	19	23.5	29	14.5
E	1	2	5	6	7	4
F	1	2	2	3	5	1
G	3	12	16.5	21	24	9
H	1	2	3.5	4	5	2
I	152	153	154	155	157	2

H



I



Annotations

Determines quartiles and inter-quartile ranges from ordered lists of data but with a minor inaccuracy.

Determines quartiles and inter-quartile ranges from data displayed in dot plots and stem-and-leaf plots.

Does not accurately determine the values of the quartiles and inter-quartile range for continuous data displayed in a cumulative frequency histogram.

Determines the quartiles and inter-quartile range from data displayed in a frequency table and frequency histogram but with a minor inaccuracy.

Algebra, measurement, geometry and statistics: Mathematics assignment

Year 10 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task/s are highlighted.

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Summary of task

This group assignment was completed at the end of a semester. It assessed several topics including quadratic equations, bivariate data, statistics and algebraic graphical representations.

In this assignment students collected data from an experiment. The assignment measured the student's understanding and the interrelationships of mathematical concepts and reasoning to draw conclusions based on the data. The students were given one week to complete the task.

Algebra, measurement, geometry and statistics: Mathematics assignment

16 cm t_{avg} = 1.5 s

12.8 cm t_{avg} = 3 s

9.6 cm t_{avg} = 2.7 s

6.4 cm t_{avg} = 2.2 s

3.2 cm t_{avg} = 1.3

~~I tested the whirlybirds~~

The whirlybirds were tested in a stairwell with a ~~fall~~ of 2.3 m, with wing lengths of 16, 12.8, 9.6, 6.4 and 3.2 cm. There was a slight crosswind for all tests, and all tests were conducted in succession on the same day.

The body dimensions were 10 cm x 8 cm, and the wing width was 1 cm

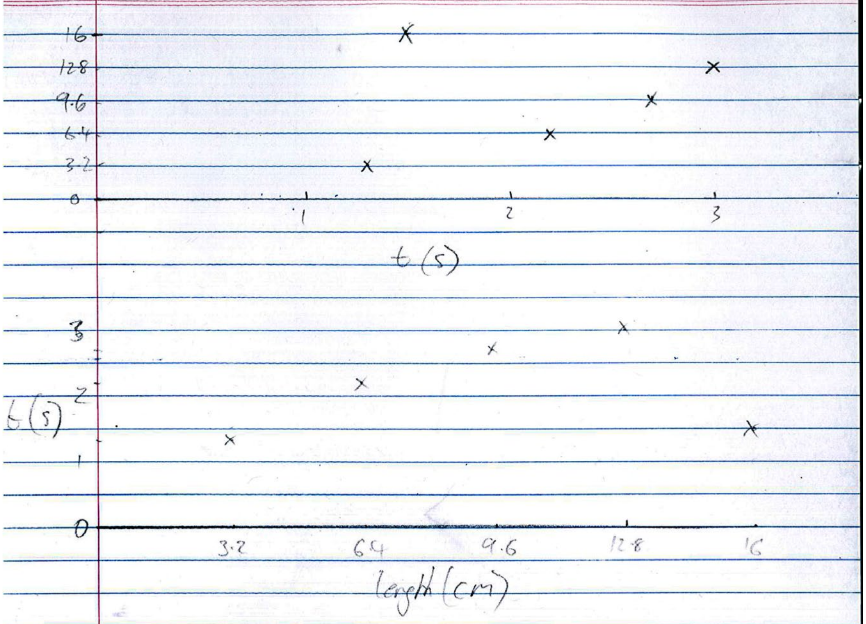
Wing ~~width~~ length (cm)	test 1	test 2	test 3	t_{avg} (s)
16	1.5	1.6	1.4	1.5
12.8	3.4	2.8	2.8	3
9.6	2.5	2.8	2.8	2.7
6.4	2.2	2.2	2.1	2.2
3.2	1.1	1.5	1.3	1.3

Annotations

Explains how experiment was conducted.

Records data obtained in the experiment.

Algebra, measurement, geometry and statistics: Mathematics assignment



It appears that once the wings reach a critical length, the performance of the whirly bird drops dramatically. Also, the smaller the wings, the more stable the flight.

Annotations

Draws graph of data obtained.

Draws conclusions based on the data obtained.

Mathematics

Year 10
Satisfactory

Algebra, measurement, geometry and statistics: Mathematics assignment

Lines	Region Max	Region Min
2	6	5
3	10	9
4	15	13
5	21	19
6	28	25
7	36	33
8	45	41
0	2	2
1	3	3

2 3 4 5
6 10 15 21
4 5 6 7
1 1 1 1

Solve for $y = ax^2 + bx + c$

$(2, 6)$ $6 = 4a + 2b + c$ (1)
$(3, 10)$ $10 = 9a + 3b + c$ (2)
$(4, 15)$ $15 = 16a + 4b + c$ (3)

(3) − 2×(1)
$3 = 8a - c$ (4)

3(1) − 2(2)
$-2 = -6a + c$ (5)

(4)+(5)
$1 = 2a$
$a = \frac{1}{2}$

(4) −
$3 = 4 - c$
$c = 1$

(1)
$6 = 2 + 2b + 1$
$2b = 3$
$b = \frac{3}{2}$

$y = \frac{x^2}{2} + \frac{3}{2}x + 1$

for $x = 3$,
$y = 4.5 + 4.5 + 1 = 10$
for $x = 2$,
$y = 2 + 3 + 1 = 6$
for $x = 1$,
$y = \frac{1}{2} + \frac{3}{2} + 1$
$= 3$

Annotations

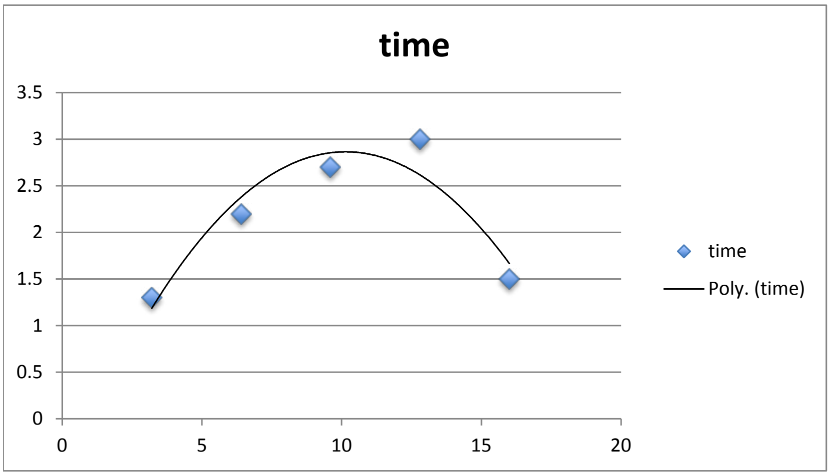
Draws a table of information from the diagrams of circles and tangents.

Solves quadratic equations to find a solution.

Substitutes into equations to find solutions.

Algebra, measurement, geometry and statistics: Mathematics assignment

length	time				
16	1.5				
12.8	3				
9.6	2.7			22.5	506.25
6.4	2.2				
3.2	1.3				



Annotations

Constructs graph from data using technology.

Algebra, measurement, geometry and statistics: Mathematics assignment

for $x=0$

$y = 0+0+1$

$= 1$

Unfortunately, the model doesn't support the ~~scenario~~ scenario where there are no tangents.

$253 = \frac{x^2}{2} + \frac{3x}{2} + 1$

$0 = \frac{x^2}{2} + \frac{3x}{2} - 252$

$x = \frac{-\frac{3}{2} + \sqrt{\frac{9}{4} + 504}}{1}, \frac{-\frac{3}{2} - \sqrt{\frac{9}{4} + 504}}{1}$

$= -\frac{3}{2} + 22.5, -\frac{3}{2} - 22.5$

$x = 21, -24$

$\therefore$ 21 tangents are needed

For an even number of tangents, the minimum number of regions is equal the maximum number of regions minus half the number of tangents.

Annotations

Draws conclusions based on data collected.

Substitutes into equation correctly to find the solution.

Draws a conclusion based on the information gathered.