Mathematics

Year 9
Above satisfactory

WORK SAMPLE PORTFOLIO

Annotated work sample portfolios are provided to support implementation of the Foundation – Year 10 Australian Curriculum.

Each portfolio is an example of evidence of student learning in relation to the achievement standard. Three portfolios are available for each achievement standard, illustrating satisfactory, above satisfactory and below satisfactory student achievement. The set of portfolios assists teachers to make on-balance judgements about the quality of their students' achievement.

Each portfolio comprises a collection of students' work drawn from a range of assessment tasks. There is no pre-determined number of student work samples in a portfolio, nor are they sequenced in any particular order. Each work sample in the portfolio may vary in terms of how much student time was involved in undertaking the task or the degree of support provided by the teacher. The portfolios comprise authentic samples of student work and may contain errors such as spelling mistakes and other inaccuracies. Opinions expressed in student work are those of the student.

The portfolios have been selected, annotated and reviewed by classroom teachers and other curriculum experts. The portfolios will be reviewed over time.

ACARA acknowledges the contribution of Australian teachers in the development of these work sample portfolios.

THIS PORTFOLIO: YEAR 9 MATHEMATICS

This portfolio provides the following student work samples:

Sample 1	Measurement: Trigonometry
Sample 2	Measurement: Wheelchair access (Pythagoras' Theorem)
Sample 3	Measurement: Tall and short (volume of a cylinder)
Sample 4	Geometry: Similar triangles
Sample 5	Probability: Probabilities
Sample 6	Number: Index laws
Sample 7	Algebra: Linear relationships
Sample 8	Measurement: Volume of a prism
Sample 9	Measurement: Surface area and volume
Sample 10	Statistics: Data displays
Sample 11	Measurement: Trigonometry and similarity in right-angled triangles
Sample 12	Measurement and geometry: Cylinder volume
Sample 13	Algebra: Coordinate geometry

Mathematics

Year 9
Above satisfactory

This portfolio of student work shows the application of the index laws to numbers (WS6) and expresses numbers in scientific notation (WS6). The student finds the distance between two points on the Cartesian plane, the gradient and midpoint of a line segment and sketches linear relationships (WS7, WS13). The student recognises the connection between similarity and trigonometric ratios (WS11) and uses Pythagoras' Theorem (WS2) and trigonometry to find unknown sides in right-angled triangles (WS1, WS11). The student uses measurement, ratio and scale factor to calculate unknown lengths in similar figures (WS4, WS11). The student calculates the areas of shapes and the volumes and surface areas of right prisms and cylinders (WS3, WS8, WS9, WS12). The student interprets and represents data in back-to-back stem-and-leaf plots and frequency histograms (WS10). The student calculates relative frequencies to estimate probabilities, lists outcomes for two-step experiments and assigns probabilities for those outcomes (WS5).

Measurement: Trigonometry

Year 9 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task are highlighted.

By the end of Year 9, students solve problems involving simple interest. They interpret ratio and scale factors in similar figures. Students explain similarity of triangles. They recognise the connections between similarity and the trigonometric ratios. Students compare techniques for collecting data in primary and secondary sources. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.

Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations. Students calculate areas of shapes and the volume and surface area of right prisms and cylinders. They use Pythagoras' Theorem and trigonometry to find unknown sides of right-angled triangles. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They construct histograms and back-to-back stem-and-leaf plots.

Summary of task

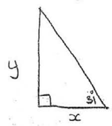
Students had completed a unit of work on the trigonometric ratios. They were given a quiz to be completed as a class test during a lesson.

Measurement: Trigonometry

Quiz 1 – Angles

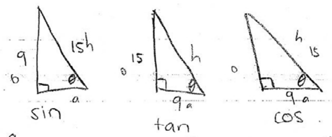
1. Consider Tan 31°. Explain as much as you can from this information. What can this tell you about the triangle?

The θ angle is 31°
You are looking at the opposite & adjacent sides of the triangle.
The triangle is right angled.
You would use Tan31° to find one of the sides of the triange (either o/a)
The triange may look like:



2. Two of the side lengths of a right angled triangle are 9 and 15. What could the reference angle be? Explain your thinking.

It could be 3 different triangles:



$\sin\theta = \frac{9}{15}$ $\quad \tan\theta = \frac{15}{9}$ $\quad \cos\theta = \frac{9}{15}$

$= \sin^{-1}(9 \div 15)$ $\quad \theta = \tan^{-1}(15 \div 9)$ $\quad \theta = \cos^{-1}(9 \div 15)$

$\theta = 36.87°$ $\quad \theta = 59.04°$ $\quad \theta = 53.13°$

- The two side lengths can be placed in three different ways, to make three different triangles
- depending on which sides the lengths are for on the triangle is what determines what the reference angle will be
- so you could be using any of the angle's ratio's (sin, cos, tan) and each one will have a different angle.

therefore the reference angle could be either 36.87° (sin), 59.04° (tan) or 53.13° (cos).

Annotations

Explains the tangent ratio in relation to the angle.

Draws and labels the sides of three possible right-angled triangles, recognising that the hypotenuse must be the longer of the two sides when using the sine and cosine ratios.

Demonstrates understanding of the use of the sine, cosine and tangent ratios.

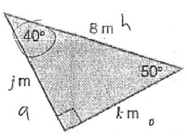
Explains the reasoning of the positioning of the sides.

Measurement: Trigonometry

Quiz 2 – Sides

1. The following answers were given by a student on a trigonometry test.

i. Find the value of k.



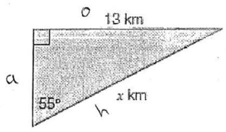
$\cos\theta = \frac{A}{H}$

$\cos 40° = \frac{k}{8}$

$8 \times \cos 40° = k$

$k = 6.13$ m

ii. Find the value of x.



$\sin\theta = \frac{O}{H}$

$\sin 55° = \frac{13}{x}$

$13 \times \sin 55° = x$

$x = 10.65$ km

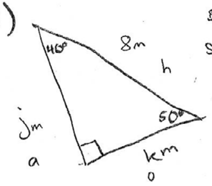
a) Explain the mistake the student has made in each question.

in Q i the student has done the correct working out but has used the wrong ratio, she used cos instead of sin. She still could have used cos but she would of had to change 50° to her reference angle.

this time in Q ii the student has got the right ratio and has done everything right up until the third step. the student has multiplied sin55° by 13 when 13 should have been divided by sin 55°.

b) Show the correct calculations and answers.

i)



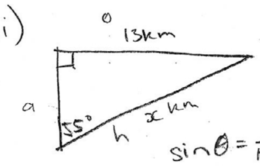
$\sin\theta = \frac{O}{H}$

$\sin 40° = \frac{k}{8}$

$8 \times \sin 40° = k$

$k = 5.14$m

ii)



$\sin\theta = \frac{O}{H}$

$\sin 55° = \frac{13}{x}$

$x = \frac{13}{\sin 55°}$

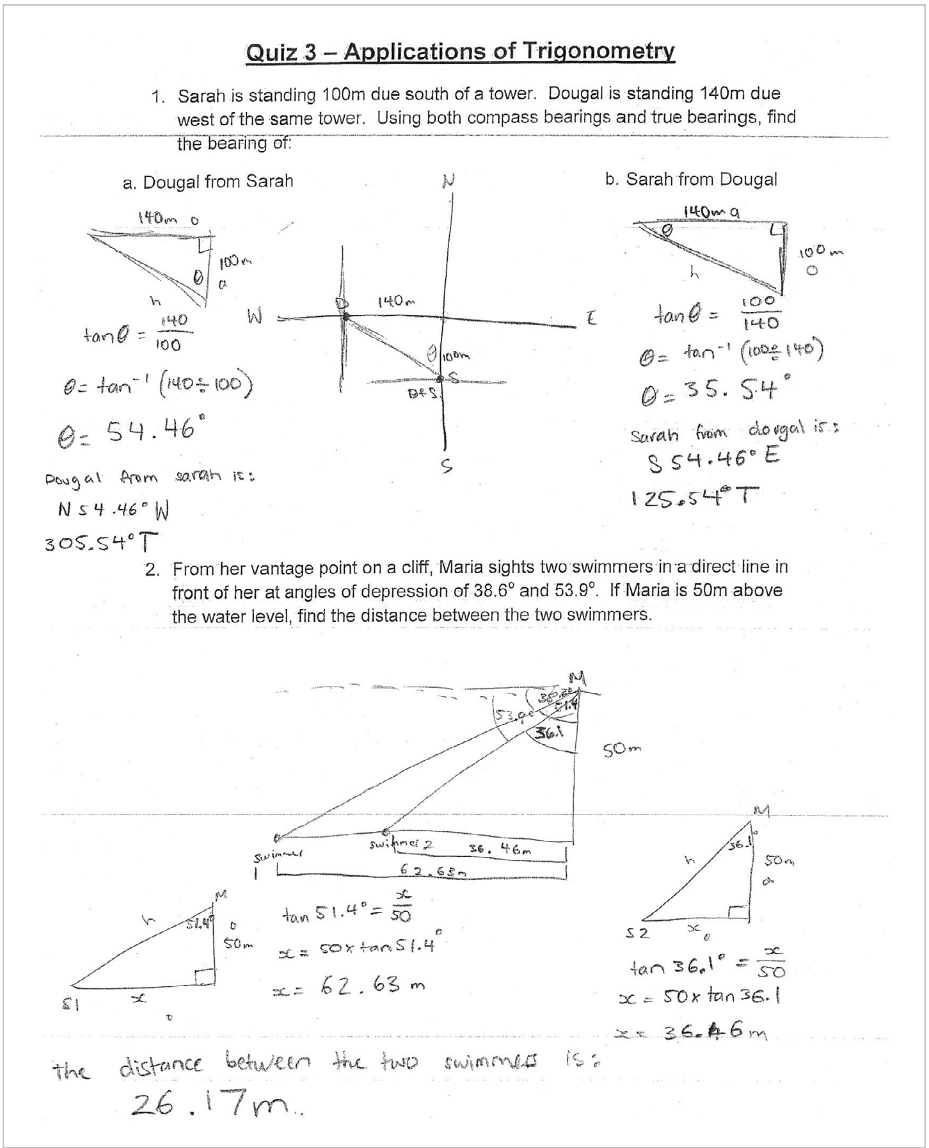
$x = 15.87$ km

Annotations

Identifies and explains the mistakes and provides correct alternatives.

Uses trigonometry to find unknown sides of right-angled triangles solving both for the hypotenuse and another side.

Measurement: Trigonometry



Quiz 3 – Applications of Trigonometry

1. Sarah is standing 100m due south of a tower. Dougal is standing 140m due west of the same tower. Using both compass bearings and true bearings, find the bearing of:

a. Dougal from Sarah

$\tan\theta = \frac{140}{100}$

$\theta = \tan^{-1}(140 \div 100)$

$\theta = 54.46^\circ$

Dougal from sarah is:

N 54.46° W

305.54°T

b. Sarah from Dougal

$\tan\theta = \frac{100}{140}$

$\theta = \tan^{-1}(100 \div 140)$

$\theta = 35.54^\circ$

Sarah from dougal is:

S 54.46° E

125.54°T

2. From her vantage point on a cliff, Maria sights two swimmers in a direct line in front of her at angles of depression of 38.6° and 53.9°. If Maria is 50m above the water level, find the distance between the two swimmers.

$\tan 51.4^\circ = \frac{x}{50}$

$x = 50 \times \tan 51.4^\circ$

$x = 62.63$ m

$\tan 36.1^\circ = \frac{x}{50}$

$x = 50 \times \tan 36.1$

$x = 36.46$ m

the distance between the two swimmers is:

26.17m.

Annotations

Calculates an appropriate angle and uses this to determine the required bearings.

Calculates each distance and then uses them to answer the question.

Measurement: Wheelchair access (Pythagoras' Theorem)

Year 9 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task are highlighted.

By the end of Year 9, students solve problems involving simple interest. They interpret ratio and scale factors in similar figures. Students explain similarity of triangles. They recognise the connections between similarity and the trigonometric ratios. Students compare techniques for collecting data in primary and secondary sources. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.

Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations. Students calculate areas of shapes and the volume and surface area of right prisms and cylinders. They use Pythagoras' Theorem and trigonometry to find unknown sides of right-angled triangles. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They construct histograms and back-to-back stem-and-leaf plots.

Summary of task

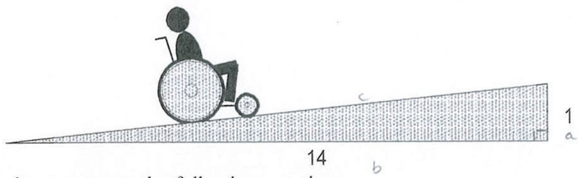
Students had completed a unit of work on Pythagoras' Theorem. They were given a worksheet with questions relating to Australian Standards Council regulations for slopes of ramps into buildings. Students completed the task as a class test during a lesson.

Measurement: Wheelchair access (Pythagoras' Theorem)

23. Wheelchair Ramps, Slopes and Accessibility

The Australian Standards Council has regulations for slopes of ramps into buildings, in order for wheelchairs to be accessible to the buildings. Such ramps must have no greater slope than 1 in 14.

By the term "1 in 14", we mean that for every 14 metres travelled horizontally (<u>not</u> actually on the ramp), we rise 1 metre. (The diagram below is not to scale.)



Use this information to answer the following question:

1. If a person effectively rises 1 metre <u>vertically</u> in moving along a 1 in 14 ramp, what is the length of the ramp? Please explain your working.

Length of ramp = c.

$a^2+b^2=c^2$

$c=\sqrt{1^2+14^2}$

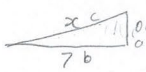
$=14.04$ units.

Using Pythagoras' theorem, the length of the ramp is calculated as it represents the hypotenuse of the right angled triangle.

2. You have been asked to work out the size and cost of a ramp for accessibility to a portable classroom at a school. The ramp must rise by a total of 0.5 m.

a) What would be the minimum length of such a ramp?

$1:14$
$\div 0.5$
$=0.5:7$



$c=\sqrt{0.5^2+7^2}$
$=7.02$ units.

∴ The minimum length of such a ramp is 7.02 units.

b) If the ramp is 1.5 m wide, and non-slip materials used in making the ramp cost \$25 per square metre, what will be the cost of the non-slip surface of the ramp? Once again, please show your working.

Non slip surface
$=1.5\times 7.02$
$=10.53\ m^2$

Cost:
10.53×25
$=\$263.17$.

Annotations

Recognises that Pythagoras' Theorem applies and uses it to determine the required length.

Uses ratio and similar triangles to determine the side lengths of the triangle and then calculates the required length using Pythagoras' Theorem.

Correctly calculates the cost, clearly showing the steps in the solution process.

Measurement: Tall and short (volume of a cylinder)

Year 9 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task are highlighted.

By the end of Year 9, students solve problems involving simple interest. They interpret ratio and scale factors in similar figures. Students explain similarity of triangles. They recognise the connections between similarity and the trigonometric ratios. Students compare techniques for collecting data in primary and secondary sources. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.

Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations. Students calculate areas of shapes and the volume and surface area of right prisms and cylinders. They use Pythagoras' Theorem and trigonometry to find unknown sides of right-angled triangles. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They construct histograms and back-to-back stem-and-leaf plots.

Summary of task

Students had completed a unit of work on surface area and volume. They were given a worksheet pertinent to this topic and asked to complete it without assistance during a lesson.

Measurement: Tall and short (volume of a cylinder)

20. "Tall and Thin" or "Short and Fat"

By taking appropriate measurements and carrying out calculations, answer the following question:

Which would hold the most:

- a cylinder made from an A4 sheet of paper, rolled so that it is "tall and thin";

OR

- a cylinder made from an A4 sheet of paper, rolled so that it is "short and fat".

Please calculate the capacity in each case, show all your working, and then answer the question: "which would hold the most?"

A4 paper:
Length: 29.6 cm
width: 20.9 cm

Tall and thin:
Circumference = 20.9 cm
$D = \frac{C}{\pi}$
$D = \frac{20.9}{\pi}$
$= 6.6527$
$r = 3.3263$
$V = \pi r^2 h$
$= \pi \times 3.3263^2 \times 29.6$
$= 1028.8793 \text{ cm}^3$
$= 1028.88 \text{ mL}$

Short and fat:
Circumference = 29.6 cm
$D = \frac{C}{\pi}$
$D = \frac{29.6}{\pi}$
$= 9.42197$
$r = 4.71098$
$V = \pi r^2 h$
$= \pi \times 4.71098^2 \times 20.9$
$= 1457.1984 \text{ cm}^3$
$= 1457.20 \text{ mL}$

The cylinder made out of an A4 sheet of paper that would hold the most is the "short and fat" cylinder, as it holds approximately 428.32 mL more than the "tall and thin" cylinder.

Annotations

Records measurements of A4 sheet.

Calculates the radii of the cylinders given their circumferences.

Correctly converts from units of volume to units of capacity.

Calculates the volume of each cylinder correct to two decimal places.

Compares capacities to determine which is the greater and calculates the size of the difference.

Mathematics

Year 9
Above satisfactory

Geometry: Similar triangles

Year 9 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task are highlighted.

> *By the end of Year 9, students solve problems involving simple interest. They interpret ratio and scale factors in similar figures. Students explain similarity of triangles. They recognise the connections between similarity and the trigonometric ratios. Students compare techniques for collecting data in primary and secondary sources. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.*
>
> *Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations. Students calculate areas of shapes and the volume and surface area of right prisms and cylinders. They use Pythagoras' Theorem and trigonometry to find unknown sides of right-angled triangles. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They construct histograms and back-to-back stem-and-leaf plots.*

Summary of task

Students had been investigating the concepts included in the study of similar triangles. They were given the task of measuring the angle of elevation of some common objects around the school, and worked in pairs to complete a short worksheet using the measurements to make a series of measurements and calculations.

Geometry: Similar triangles

Task: Work in pairs

1. Use the clinometers to measure the angles of elevation of 4 objects around the school. Eg basketball stand, flagpole, street light, building, tree, football goal posts. ***Record the angles.*** Each person is to choose 4 objects that are **different from their partner's** objects.

2. Measure the distance from where you were standing to the base of the object whose angle of elevation you measured. ***Record the distances.***

3. Measure your own height from floor to eye level. ***Record the height.***

4. In the classroom, draw four right-angled triangles, each with a base length of 5 cm and an angle that corresponds to each of the angles of elevation that you measured outside.

5. Calculate the height of each object using the similar triangles

Object	Angle of elevation	Distance to object
light pole	18°	9m
tall tree out front	46°	27m
Canteen Door	8°	8m
Sliding doors (canteen)	6°	9m

Your height to eye level 145cm

What to hand in:

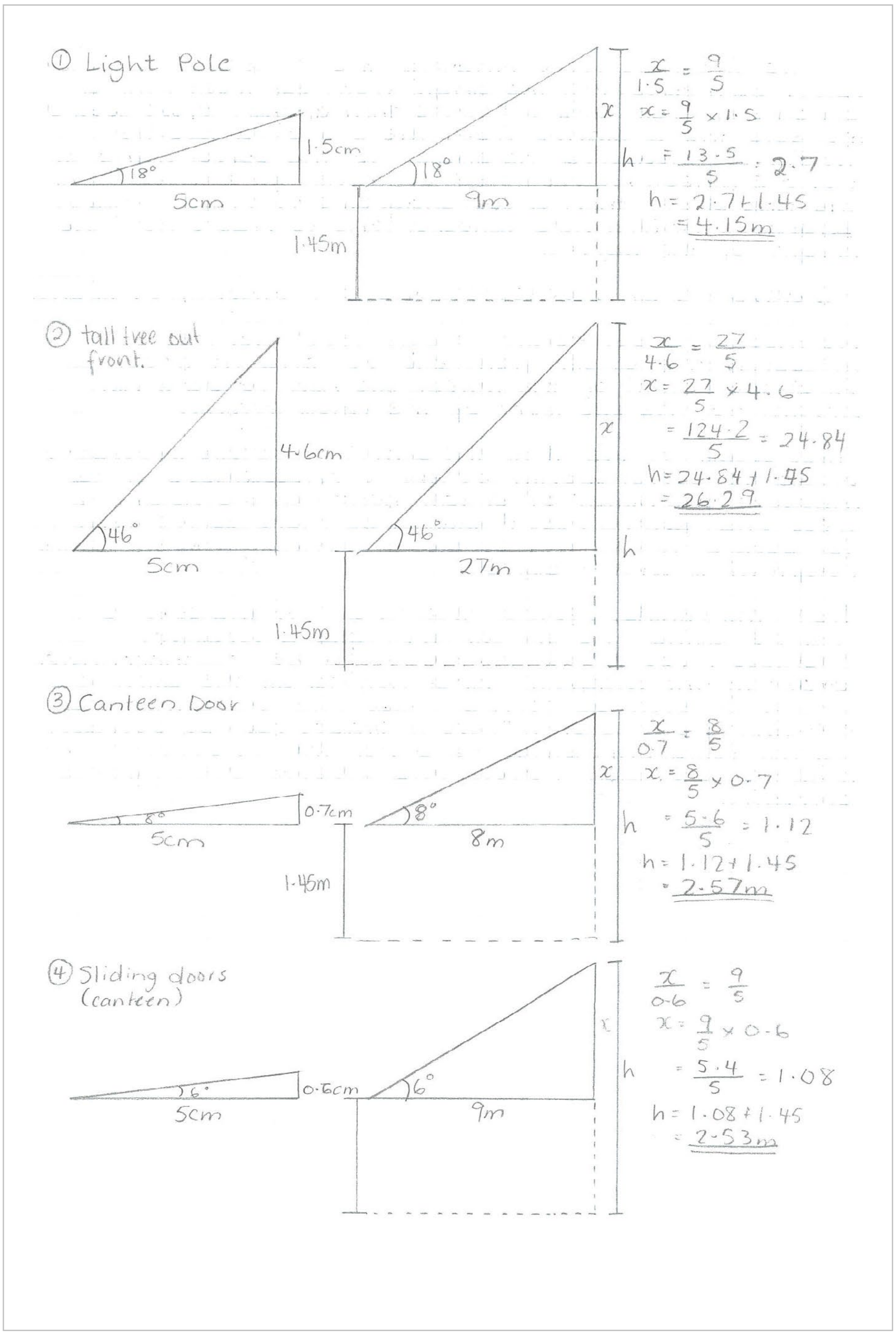
1. This sheet with your measurements included.
2. **Introduction** – a paragraph to explain what you are doing or finding out in this D.I. and how you went about the task.
3. **Mathematical procedures** – all diagrams and calculations.
4. **Analysis** – answer the questions below in well-written sentences.
   - Why did you have to measure your height?
   - List 3 ways in real life that this similar triangle procedure would be useful.
5. **Conclusion** – a paragraph to explain what you found out, where you could have made mistakes and how these mistakes could have been avoided.

- Communication – is your work easily understood, do your sentences make sense and have no spelling or grammar mistakes?
- Presentation – is your work neat and tidy? Are your diagrams large enough with names and labels? Are all your calculations clearly set out including formula used and working out done?

Annotations

Records angles of elevation, own height and distances as measured.

Geometry: Similar triangles



Annotations

Uses similar triangles to calculate heights illustrated by diagrams, clearly demonstrating understanding of the comparisons and ratios involved.

Clearly communicates mathematical processes, including the need to add observer's height to the calculated length of the triangle side x to find the final height of the object.

Mathematics

Year 9

Above satisfactory

Geometry: Similar triangles

In this D.I., we were finding the height of a certain object by measuring the angle from eye level with a clinometer from any distance. Then afterward, we would measure the distance from the standing position (where we measured the angle) to the object using a trundle wheel. We recorded our measurement results on the task sheet then drew diagrams of right-angled triangles and made calculations to figure out the height of the object.

We measured our height to eye level because we measured the angle from eyelevel and in order to find the height of the object, we had to add our height to the eye level of the calculations.

Three ways in life that the similar triangle procedure would be useful would be for when you needed to measure the height of a tall building, the height of trees and powerlines. It would be more convenient for people to use it instead of actually measuring the height of a certain object.

From my results, I found the height of my selected objects using the similar triangles procedure. Mistakes I could have made may be from inaccurately drawing the diagram and getting all the calculations wrong or even gathering inaccurate data, making everything else wrong. These mistakes can be avoided by carefully checking the data that I collected or making sure that I drew and labelled the digrams correctly.

Annotations

Clearly explains why the height of the eye level of the person must be taken into account by referring to the measurement process.

Identifies possible sources of error when measurements are made and/or used in calculations.

Probability: Probabilities

Year 9 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task are highlighted.

> *By the end of Year 9, students solve problems involving simple interest. They interpret ratio and scale factors in similar figures. Students explain similarity of triangles. They recognise the connections between similarity and the trigonometric ratios. Students compare techniques for collecting data in primary and secondary sources. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.*
>
> *Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations. Students calculate areas of shapes and the volume and surface area of right prisms and cylinders. They use Pythagoras' Theorem and trigonometry to find unknown sides of right-angled triangles. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They construct histograms and back-to-back stem-and-leaf plots.*

Summary of task

Students had been collecting data from experiments and using their data to investigate probabilities. Students were given the objects to complete this task in a 15-minute time period.

Mathematics

Year 9
Above satisfactory

Probability: Probabilities

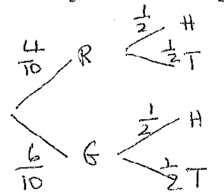
Probabilities

You have a bag of 10 balls containing 4 red ball and 6 green balls. You also have a coin which you can toss to get a head or a tail. You are going to pick a ball from your bag and then toss a coin 20 times.

Record your results in the table below.

	Colour of ball R or G	Toss of the coin H or T
1	R	T
2	G	T
3	G	H
4	R	T
5	G	H
6	G	T
7	R	T
8	G	H
9	G	H
10	R	T
11	G	H
12	R	T
13	G	H
14	G	T
15	R	H
16	G	H
17	R	T
18	G	H
19	R	H
20	R	H

List below all the possible results from choosing a ball and tossing a coin



RH	|||	3
RT	|||| |	6
GH	|||| |||	8
GT	|||	3

1. How many times would you expect to choose a green ball and toss a tail? $\frac{6}{10} \times \frac{1}{2} \times 20 = 6$
2. How many times would you expect to choose a red ball and toss a head? $\frac{4}{10} \times \frac{1}{2} \times 20 = 4$
3. Did your results differ from what you would expect? yes

Can you explain why there might be a difference? If you calculate the expected probalities you can see what you would expect to get for G T and R H. Mine are different because there is always a difference between what you expect and what you get in reality.

Annotations

Lists possible outcomes of the experiment using a tree diagram and calculates the theoretical probability for each outcome as a percentage.

Summarises their results using a frequency table.

Calculates expected frequencies using theoretical probabilities.

Demonstrates an understanding of the difference between relative frequencies obtained from an experiment and theoretical probabilities.

Number: Index laws

Year 9 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task are highlighted.

By the end of Year 9, students solve problems involving simple interest. They interpret ratio and scale factors in similar figures. Students explain similarity of triangles. They recognise the connections between similarity and the trigonometric ratios. Students compare techniques for collecting data in primary and secondary sources. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.

Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations. Students calculate areas of shapes and the volume and surface area of right prisms and cylinders. They use Pythagoras' Theorem and trigonometry to find unknown sides of right-angled triangles. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They construct histograms and back-to-back stem-and-leaf plots.

Summary of task

Students had been revising index laws and applying them to numbers. They had investigated the use of scientific notation in various contexts. Students were asked to complete this quick quiz in a 15-minute time period.

Number: Index laws

Index laws and Numbers

1. Answer the following questions

Question	Answer	Question	Answer
1. $2^3 x 2^5$=	$=2^8$ $=256$	2. $2^6 \div 2^4$=	2^2 $=4$
3. $4^2 x 4^1$=	$=4^3$ $=64$	4. $7^7 \div 7^5$=	$=7^2$ $=49$
5. $6^1 x 6^1$=	$=6^2$ $=36$	6. $8^4 \div 8^4$=	$=8^0$ $=1$
7. $(2^3)^2$=	$=2^6$ $=64$	8. 10^0=	$=1$
9. $2(3^0)^2$=	$=2\times3^0$ $=2$	10. $2^3 \div 2^5$=	$=2^{-2}$ $=\frac{1}{2^2}=\frac{1}{4}$
11. $25^{\frac{1}{2}}$=	$=\sqrt{25}$ $=5$	12. $16^{\frac{1}{2}} x 16^{\frac{1}{2}}$=	$=16^{\frac{1}{4}}$ $=\sqrt[4]{16}=2$

2. Express the following numbers in scientific notation:

Question	Answer	Question	Answer
1. 100	1.0×10^2	2. 5010	$=5.01\times10^3$
3. 210000	$=2.1\times10^5$	4. 7567	$=7.567\times10^3$
5. 0.0025	$=2.5\times10^{-3}$	6. 0.00000012	$=1.2\times10^{-7}$
7. 32654	$=3.2654\times10^4$	8. 0.000003652	3.652×10^{-6}
9. 10001000	~~=10001×10⁷~~ $=1.0001\times10^7$	10. 0.001000356	$=1.000356\times10^{-3}$

3. Why is it necessary to write numbers in scientific notation? Can you give examples?

Scientific notation makes it possible to write very large and very small numbers quickly and easily. For example the very small numbers in nanotechnology or very large numbers in space exploration

Annotations

Uses index laws to correctly evaluate all numerical expressions, leaving answers in simplest form.

Correctly identifies the positive and negative powers of 10 and expresses numbers in scientific notation.

Explains the purpose of writing numbers in scientific notation and provides a relevant context for their use.

Algebra: Linear relationships

Year 9 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task are highlighted.

By the end of Year 9, students solve problems involving simple interest. They interpret ratio and scale factors in similar figures. Students explain similarity of triangles. They recognise the connections between similarity and the trigonometric ratios. Students compare techniques for collecting data in primary and secondary sources. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.

Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations. Students calculate areas of shapes and the volume and surface area of right prisms and cylinders. They use Pythagoras' Theorem and trigonometry to find unknown sides of right-angled triangles. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They construct histograms and back-to-back stem-and-leaf plots.

Summary of task

Students had completed a unit of work on linear relationships. They had investigated the gradient and midpoint of the interval joining two points and the distance between those two points on the Cartesian plane. Students were given a series of questions on the topic and completed the task as a test in class.

Algebra: Linear relationships

Number and Algebra

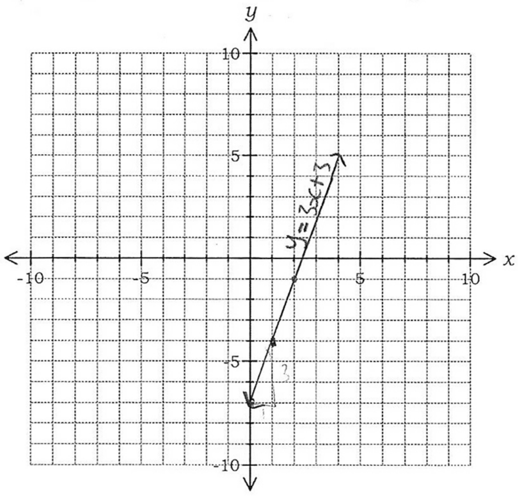
- Answer all questions neatly in the spaces provided.
- **Show all working** where appropriate.
- If necessary, round all answers to **2 decimal places** unless stated otherwise.
- **Calculator allowed.**

Question 1

Plot the line represented by the points in the following table on the axes provided below.

x	0	1	2	3	4
y	-7	-4	-1	2	5

$y = 3x + 3$ 3   3   3   3



Annotations

Constructs line in correct position using the ordered pairs provided but has incorrect equation labelled on line.

Algebra: Linear relationships

Question 2

(a) The tables below represent linear relationships. How can you tell?

(i)

x	0	1	2	3	4
y	4	9	14	19	24

1 1 1 1

5 5 5 5

(ii)

x	1	2	3	4	5
y	10	7	4	1	-2

0 1 1 1 1

13 -3 -3 -3 -3

Because they both x and y on both tables go up the same amount each time.

(b) Determine the rule between x and y for the tables in (a).

(i) $\boxed{y = 5x + 4}$

$\frac{\text{rise}}{\text{run}} = \frac{5}{1} = 5$

$4 = 5 \times 0 + c$

$4 = 0 + c$

$4 = c$

(ii) $\boxed{y = -3x + 13}$

$\frac{\text{rise}}{\text{run}} = \frac{-3}{1} = -3$

$10 = -3 \times 1 + c$

$10 = -3 + c$

$10 + 3 = c$

$13 = c$

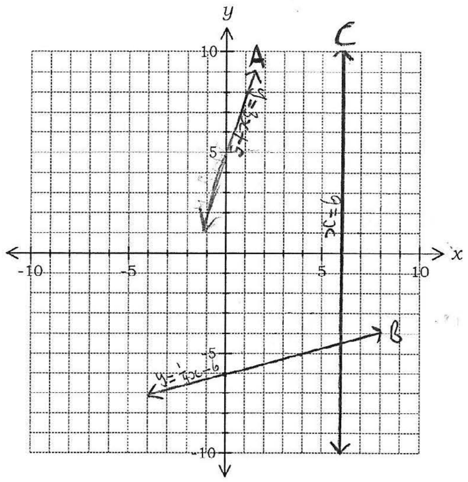
Question 3

On the axes below, plot the following lines, labelling each one.

A: a line that has a gradient of 3 and a y-intercept at (0, 5).

B: the line $y = \frac{1}{4}x - 6$.

C: the line $x = 6$.



Annotations

Demonstrates understanding of linear relationships. Identifies the 'common difference' for each table of values and explains why the relationships are linear.

Determines the rule for each table of values.

Plots linear relationships using correct intercepts and correct gradients.

Algebra: Linear relationships

Question 4

Determine the equations of the following lines. Show all working.

(a) The line with a gradient of $\frac{1}{2}$ with a y-intercept of 6.

$y = \frac{1}{2}x + 6$

(b) The line that has a gradient of 4 and passes through the point $(\overset{x}{2}, \overset{y}{3})$.

$y = 4x + c$

$3 = 4 \times 2 + c$

$3 = 8 + c$

$3 - 8 = c$

$-5 = c$

$y = 4x - 5$

(c) The line that passes through the points $(\overset{x^1}{2}, \overset{y^1}{5})$ and $(\overset{x^2}{-3}, \overset{y^2}{-10})$.

$\underline{y = mx + c}$

$m = \frac{-10 - 5}{-3 - 2} = \frac{-15}{-5} = 3$

$\underline{y = 3x + c}$

$c = 5 = 3 \times 2 + c$

$5 = 6 + c$

$5 - 6 = c$

$-1 = c$

$\underline{\underline{y = 3x - 1}}$

Annotations

Determines the equations of lines from a variety of given information.

Algebra: Linear relationships

Question 5

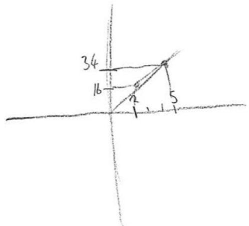
Tahleah babysits to earn money. For all her clients she charges an hourly fee and also an additional one off fee for each babysitting job.

(a) If for a 2 hour babysitting job she charges \$16 and for a 5 hour babysitting job she charges \$34, determine the rule that she uses to calculate the amount she charges, $\$C$, for each babysitting job of h hours.

$\boxed{C = 6h + 4}$

$\frac{\text{Rise}}{\text{run}} = \frac{18}{3} = 6$

$16 - 6 \times 2 = 4$



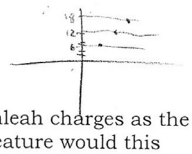
(b) Use your rule from (a) to calculate how much Tahleah would charge for a three and a half hour babysitting job.

$C = 6 \times 3\frac{1}{2} + 4$

$C = \$25$

(c) (i) With reference to your rule, state the amount the Tahleah charges per hour. If you graphed the line, what feature would this value represent?

She charges \$6 per hour. If this was graphed, the lines points would go up by 6 each time. The 6 is the gradient.



(ii) With reference to your rule, state the amount the Tahleah charges as the additional fee per job. If you graphed the line, what feature would this value represent?

The additional fee is \$4. If this was graphed, it would represent the y-intercept.

Annotations

Recognises that the relationship is linear and determines a rule using the given information.

Uses the rule from previous question to determine the value.

Explains what the value the gradient and the y-intercept of the line represent in the context of the problem.

Algebra: Linear relationships

Question 6

Determine the co-ordinates of the midpoint between the points (3, -7) and (5, 3).

Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{3+5}{2}, \frac{-7+3}{2}\right)$

$= \left(\frac{8}{2}, \frac{-4}{2}\right)$

$= (4, -2)$

$: (4, -2)$

Question 7

Determine the distance between the points (3, 9) and (6, 4), giving your answer to 2 decimal places.

Distance $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(6-3)^2 + (4-9)^2}$

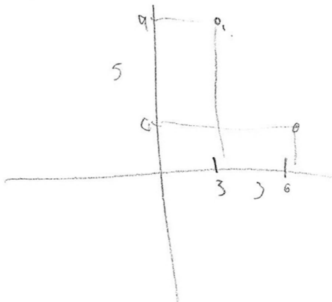
$\sqrt{\ } =$ square root

$= \sqrt{(3)^2 + (-5)^2}$

$= \sqrt{9 + 25}$

$= \sqrt{34}$

$= 5.83$



Annotations

Uses the midpoint formula to determine the coordinates of the midpoint of an interval on the Cartesian plane.

Uses the distance formula to determine the distance between two points on the Cartesian plane.

Algebra: Linear relationships

Question 8

During a sailing competition all of the boats' positions are taken relative to a buoy (ie. the buoy has co-ordinates (0, 0)). A few minutes into the competition, a boat at (3, 7) launches a distress flare. A rescue boat, positioned at (-4, -5), sees the flare and sets out immediately to assist them. [All units are in kilometres.]

(a) How far must the rescue boat travel to reach the distressed boat?

$$\sqrt{(-4-3)^2 + (-5-7)^2}$$

$$\sqrt{(-7)^2 + (-12)^2}$$

$$\sqrt{49 + 144}$$

$$\sqrt{193} = 13.89 \text{ Km}$$

(b) Exactly halfway to the distressed boat the rescue boat passed a second boat that needed assistance. They instructed this boat to drop anchor and said they would return to them once they had seen to the first distress signal. Determine the co-ordinates of the second troubled boat.

$$\left(\frac{3-4}{2}, \frac{7-5}{2}\right)$$

$$\left(\frac{-1}{2}, \frac{-2}{2}\right)$$

$$(-0.5, -1)$$

$$(-0.5, -1).$$

Annotations

Recognises that the distance formula is required and correctly substitutes all values to obtain the required distance.

Recognises that the midpoint formula is required and correctly substitutes all values but makes a minor error.

Mathematics

Year 9
Above satisfactory

Measurement: Volume of a prism

Year 9 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task are highlighted.

By the end of Year 9, students solve problems involving simple interest. They interpret ratio and scale factors in similar figures. Students explain similarity of triangles. They recognise the connections between similarity and the trigonometric ratios. Students compare techniques for collecting data in primary and secondary sources. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.

Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations. Students calculate areas of shapes and the volume and surface area of right prisms and cylinders. They use Pythagoras' Theorem and trigonometry to find unknown sides of right-angled triangles. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They construct histograms and back-to-back stem-and-leaf plots.

Summary of task

Students had completed a unit of work on volume and surface area. The activity involved a real-world problem in which they were given the volume of a cuboid and asked to determine appropriate dimensions given a particular relationship between them. Students were given 10 minutes to complete the task in class.

Mathematics

Year 9

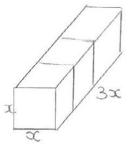
Above satisfactory

Measurement: Volume of a prism

Task Three: Volume of Prisms

A juice manufacturing company wishes to change the packaging of their 1 litre fruit juice products. Research has shown the most appealing dimensions of a cuboid are in the ratio of 1:1:3.

Is it possible to have a cuboid with a ratio of sides of 1:1:3 which contains exactly 1 litre of liquid? Explain.



A cuboid with a ratio of 1:1:3 would be the same as 3 cubes placed together.

Total Volume = $3x^3$ (where x is length of side of cube)

$1L = 1000 \text{ cm}^3$

$3x^3 = 1000$

$x = \sqrt[3]{\frac{1000}{3}}$

Has a infinite number of decimal places

$\therefore$ It is impossible to have a cuboid with a ratio of sides of 1:1:3 which contains exactly 1 litre of liquid.

Annotations

Chooses an efficient approach to the problem using knowledge of algebra.

Correctly converts litres to cubic centimetres.

Provides an answer to the problem and clearly explains reasoning in mathematical terms.

Mathematics

Year 9
Above satisfactory

Measurement: Surface area and volume

Year 9 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task are highlighted.

> *By the end of Year 9, students solve problems involving simple interest. They interpret ratio and scale factors in similar figures. Students explain similarity of triangles. They recognise the connections between similarity and the trigonometric ratios. Students compare techniques for collecting data in primary and secondary sources. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.*
>
> *Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations. Students calculate areas of shapes and the volume and surface area of right prisms and cylinders. They use Pythagoras' Theorem and trigonometry to find unknown sides of right-angled triangles. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They construct histograms and back-to-back stem-and-leaf plots.*

Summary of task

Students had completed a unit of work on volume and surface area. This activity involved determining the dimensions of a cylinder with a capacity of one litre and then using the dimensions to calculate the surface of the cylinder. Students were given 10 minutes to complete the task in class.

Mathematics

Year 9
Above satisfactory

Measurement: Surface area and volume

Task 4 Surface Area and Volume

Determine the dimensions (height and radius) of a cylinder that would have a capacity of one litre. Use these dimensions to calculate the surface area of your cylinder.

1. Relevant calculations showing how you have determined the dimensions of the cylinder
2. A labelled 3D drawing/sketch of the cylinder
3. Relevant calculations for determining the surface area of the cylinder

① $V = \pi r^2 h$ and $1L = 1000\,cm^3$

$r^2 h = 1000$

let $r = 6cm$

$\pi r^2 h = 1000$

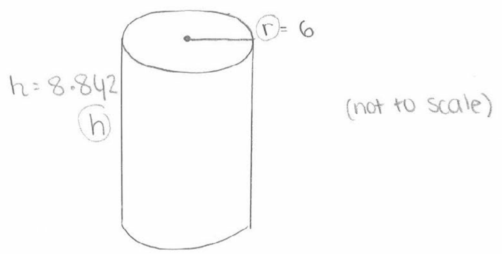
$\pi 6^2 h = 1000$

$\pi 36 h =$

$h = \frac{1000}{36\pi} = 8.842\ cm$

$\therefore$ radius $6cm$, hight $8.842\ cm$

②



③ Total Surface Area (TSA) $= 2\pi r^2 + 2\pi r h$

$= 2\pi 36 + 2\pi 6 \times \frac{1000}{36\pi}$

$= \underline{559.53\,cm^2}$

Annotations

Correctly converts litres to cubic centimetres.

Sets up an appropriate equation that can be solved to find the height of the cylinder and works through the solution with exact values to obtain the height.

Draws a cylinder and labels it with the dimensions obtained in the previous part of the task.

Uses a formula to calculate the total surface area of the cylinder with reasonable accuracy as rounding does not occur until the final calculation.

Mathematics

Year 9
Above satisfactory

Statistics: Data displays

Year 9 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task are highlighted.

> *By the end of Year 9, students solve problems involving simple interest. They interpret ratio and scale factors in similar figures. Students explain similarity of triangles. They recognise the connections between similarity and the trigonometric ratios. Students compare techniques for collecting data in primary and secondary sources. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.*
>
> *Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations. Students calculate areas of shapes and the volume and surface area of right prisms and cylinders. They use Pythagoras' Theorem and trigonometry to find unknown sides of right-angled triangles. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They construct histograms and back-to-back stem-and-leaf plots.*

Summary of task

Students had completed a unit of work on displaying data over a two-week period. In this activity students were asked to represent the given data in a back-to-back stem-and-leaf plot and frequency histograms. The activity was given as a class test to be completed in a lesson.

Statistics: Data displays

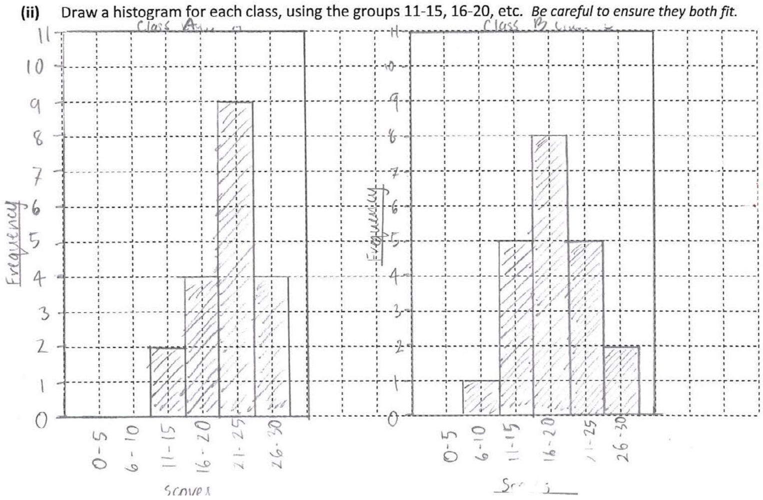
1 The data sets below show the marks scored by two classes in a class test (out of 30).

Class A	25	21	29	22	25	23	17	21	19	22	28	15	20
	27	23	20	21	14	27							
Class B	22	19	18	26	15	18	20	25	18	19	24	23	27
	11	18	14	9	20	15	21	13					

(i) Draw an **ordered** back-to-back stem-and-leaf plot to show the two classes' results.
The blank space is for working.

Class A		Class B
	0	9
9 7 5 4	1	1 3 4 5 5 8 8 8 8 9 9
9 8 7 7 5 5 3 3 2 2 1 1 1 0 0	2	0 0 1 2 3 4 5 6 7

(ii) Draw a histogram for each class, using the groups 11-15, 16-20, etc. *Be careful to ensure they both fit.*



Annotations

Constructs the stem but does not consider splitting the stem into class intervals.

Constructs an ordered back-to-back stem-and-leaf plot showing all data values from smallest to largest on each side of the stem.

Constructs frequency histograms to represent the data.

Labels both axes correctly.

Measurement and geometry: Trigonometry and similarity in right-angled triangles

Year 9 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task are highlighted.

By the end of Year 9, students solve problems involving simple interest. They interpret ratio and scale factors in similar figures. Students explain similarity of triangles. They recognise the connections between similarity and the trigonometric ratios. Students compare techniques for collecting data in primary and secondary sources. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.

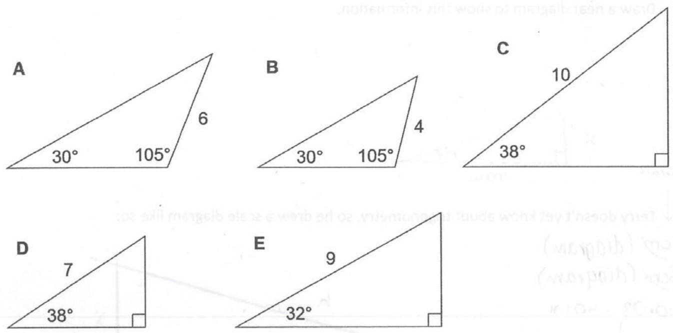
Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations. Students calculate areas of shapes and the volume and surface area of right prisms and cylinders. They use Pythagoras' Theorem and trigonometry to find unknown sides of right-angled triangles. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They construct histograms and back-to-back stem-and-leaf plots.

Summary of task

Students had completed a unit of work on trigonometry, including links to the topic of similarity that was studied earlier. In this activity, students were asked to apply their knowledge of similarity and trigonometry and apply the links between the two. The activity was given as a class test in 20 minutes.

Measurement and geometry: Trigonometry and similarity in right-angled triangles

1 Consider the following triangles.



(i) Are triangles A and B similar? Explain.

Yes, because two of their angles are the same.

(ii) Are triangles C and D similar? Explain.

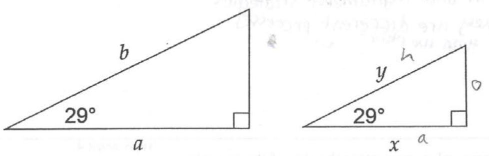
Yes, because two of their angles are the same

(iii) Are triangles D and E similar? Explain.

No, because only one of their angles ~~are~~ the same

2 The two triangles shown are similar.

SOH CAH TOA



Give **two** reasons why $\frac{a}{b} = \frac{x}{y}$.

-These triangles are similar and therefore the sides will equal the same

- $\cos 29° = \frac{x}{y} = \frac{a}{b}$

Annotations

Understands the concept of similarity and is able to explain why triangles are or are not similar.

Recognises that similarity is relevant.

Uses trigonometry to explain the equivalence of the two ratios.

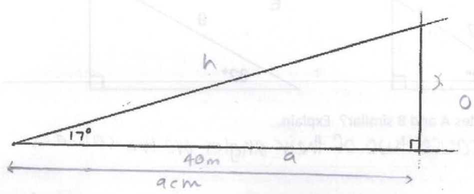
Measurement and geometry: Trigonometry and similarity in right-angled triangles

3 Terry wanted to find the height of his school's flagpole.

Having walked 40m from its base (on level ground), he measured the angle from the ground to the top of the flagpole to be 17°.

(i) Draw a neat diagram to show this information.

(ii) Terry doesn't yet know about trigonometry, so he drew a scale diagram like so:



Using a ruler, show the working Terry used to find the height of the flagpole.

40m = 9cm (diagram)
x m = 3cm (diagram)

~~9:3~~ 0.09 : 0.03 = 40 : x
×444.4̇ ×444.4̇
40 : x
x = 0.03 × 444.4̇
= 13.3̇ m

(iii) Now do your own working using trigonometry and your own diagram to find the height of the flagpole.

$\tan 17° = \frac{x}{40}$
$x = 40 \tan 17°$
$= 12.2292...$
$\doteqdot 12.23$ m

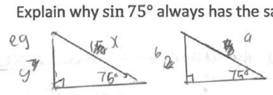
(iv) Why do the two approaches above give similar answers?

Because they are both logical and legitament strategies to find the height, ~~therefore~~ they are different processes and one is more accurate than the other

4 Explain why sin 75° always has the same value, no matter the size of the triangle.

All right angled triangles with one angle that is 75° are SIMILAR and the angles don't change.

eg



$\sin 75° = \frac{x}{y} = \frac{a}{b}$

it is a decimal and to get sin 75° you need to divide the ~~hypotenuse~~ opposite side by the hypotenuse so even with different sized triangles, the fractions can always be simplified to it's simplest form (sin 75) no matter what the numerator / denominator are or the length of side.

Annotations

Represents mathematical information given in words in diagrammatic form.

Chooses and understands an appropriate method to solve the problem but uses an inaccurate measurement.

Uses the correct trigonometric ratio to set up an equation and uses a familiar procedure to obtain the correct answer.

Uses the connection between similarity and trigonometry to explain the constancy of a trigonometric ratio.

Mathematics

Year 9
Above satisfactory

Measurement: Cylinder volume

Year 9 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task are highlighted.

> *By the end of Year 9, students solve problems involving simple interest. They interpret ratio and scale factors in similar figures. Students explain similarity of triangles. They recognise the connections between similarity and the trigonometric ratios. Students compare techniques for collecting data in primary and secondary sources. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.*
>
> *Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations. Students calculate areas of shapes and the volume and surface area of right prisms and cylinders. They use Pythagoras' Theorem and trigonometry to find unknown sides of right-angled triangles. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They construct histograms and back-to-back stem-and-leaf plots.*

Summary of task

Students had completed a section of work on cylinders. The investigation to find the volume of cylinders was given as an assignment to be completed over a week.

Mathematics

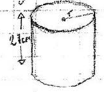
Year 9
Above satisfactory

Measurement: Cylinder volume

Cylinder Volume Investigation 04/09/12

Part A
I predict that the cylinder with a circumference of 30cm and a height of 21cm will have the largest volume.

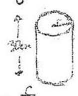
1. Cylinder A



$r = \frac{c}{2\pi}$
$r = \frac{30}{2\pi}$
$r = 4.77cm$

$V = \pi r^2 h$
$V = \pi \times 4.77^2 \times 21$
$V = 1501.09cm^3$

Cylinder B



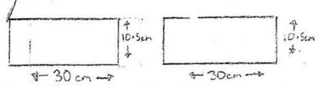
$r = \frac{c}{2\pi}$
$r = \frac{21}{2\pi}$
$r = 3.34cm$

$V = \pi r^2 h$
$V = \pi \times 3.34^2 \times 30$
$V = 1051.39 cm^3$

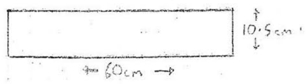
2. Cylinder A was the biggest cylinder, the volume being 449.70cm³ larger. This proves my theory from the start to be correct.

Part B
1. a) To create a larger cylinder, I could cut the paper in half length ways and join the ends together. This will create a cylinder with a circumference twice the size of the old cylinder.

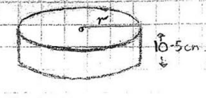
2.



=



=



3. Paper Sheet
$l = 60cm$
$w = 10.5cm$

Cylinder
$c = 60cm$
$h = 10.5cm$

4. $r = \frac{c}{2\pi}$
$r = \frac{60}{2\pi}$
$r = 9.55cm$

$V = \pi r^2 h$
$V = \pi \times 9.55^2 \times 10.5$
$V = 3008.47cm^3$

5. I believe that it is possible to create a 10L cylinder with the given sheet of paper. To do so I could continue to cut the paper in half lengthways and join the ends so that the circumference continues to double in size.

Part C
1. width = original width ÷ 4
$w = \frac{21}{4}$
$w = 5.25$

2.

Number of strips	Height cm	Circumference cm	Radius cm	Volume (cm³)	Volume (L)
4	5.25	120	19.10	6016.94	6.02
7	3	210	33.92	10526.5	10.52
9	2.33	270	42.97	13515.64	13.52
20	1.05	600	95.49	30078.42	30.08
21	1	630	100.27	31585.90	31.59
500	0.04	15000	2387.32	716197.24	716.2

3. In conclusion, it can be found that the volume of a cylinder will increase if you halve its height and double its length. Thus, its volume will infinitely increase.

Annotations

Manipulates the formula for the circumference of a circle to calculate the radius of a created cylinder.

Calculates the volume of cylinders using the formula.

Uses cubic units to describe volumetric measure.

Explains reasoning clearly and logically.

Understands the theoretical process of cutting the sheet of paper into ever thinner strips to produce a cylinder of larger and larger volume each time.

Algebra: Coordinate geometry

Year 9 Mathematics achievement standard

The parts of the achievement standard targeted in the assessment task are highlighted.

> *By the end of Year 9, students solve problems involving simple interest. They interpret ratio and scale factors in similar figures. Students explain similarity of triangles. They recognise the connections between similarity and the trigonometric ratios. Students compare techniques for collecting data in primary and secondary sources. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.*
>
> *Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations. Students calculate areas of shapes and the volume and surface area of right prisms and cylinders. They use Pythagoras' Theorem and trigonometry to find unknown sides of right-angled triangles. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They construct histograms and back-to-back stem-and-leaf plots.*

Summary of task

Students had completed a unit of work on coordinate geometry. They were given this revision task to complete in class.

Algebra: Coordinate geometry

Coordinate Geometry Revision Sheet

Question 1

A(3, 7), B(-1, 10)

a) $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
$= \sqrt{(-1-3)^2 + (10-7)^2}$
$= \sqrt{(-4)^2 + 3^2}$
$= \sqrt{25}$
$= 5$

b) mdp. $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
$= \left(\frac{3-1}{2}, \frac{7+10}{2}\right)$
$= \left(\frac{2}{2}, \frac{17}{2}\right)$
$= (1, 8.5)$

c) slope $= \frac{y_2-y_1}{x_2-x_1}$
$= \frac{10-7}{-1-3}$
$= -\frac{3}{4}$

$y = mx + c$
$7 = (-\frac{3}{4} \times 3) + c$
$7 = -\frac{9}{4} + c$
$7 + \frac{9}{4} = c$
$c = \frac{37}{4}$

$\therefore$ equation of the line:
$y = -\frac{3}{4}x + \frac{37}{4}$

(4, -3)

d) $y = \frac{4}{3}x + c$
$-3 = \frac{4}{3}(4) + c$
$-3 = \frac{16}{3} + c$
$-3 - \frac{16}{3} = c$
$-\frac{25}{3} = c$

$\therefore$ equation of the line:
$y = \frac{4}{3}x - \frac{25}{3}$

Annotations

Applies midpoint formula to given coordinates to calculate midpoint of an interval.

Applies distance formula to given coordinates to calculate distance between points.

Applies gradient formula to given coordinates to calculate slope of an interval.

Algebra: Coordinate geometry

Question 2

$C(2a, -3a)$, $D(5a, a)$

a) $d = \sqrt{(5a-2a)^2 + (a+3a)^2}$

$= \sqrt{(3a)^2 + (4a)^2}$

$= \sqrt{9a^2 + 16a^2}$

$= \sqrt{25a^2}$

$= 5a$

b) midp $= \left(\frac{2a+5a}{2}, \frac{-3a+5a}{2}\right)$

$= \left(\frac{7a}{2}, \frac{2a}{2}\right)$

$= \left(\frac{7a}{2}, a\right)$

c) $m = \frac{a+3a}{5a-2a}$

$= \frac{4a}{3a}$

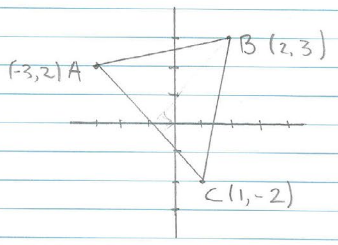
$= \frac{4}{3}$

Annotations

Applies distance, midpoint and gradient formulas to a pair of coordinates defined algebraically, simplifying answers where appropriate.

Algebra: Coordinate geometry

Question 3



a) d of AB $= \sqrt{(2+3)^2 + (3-2)^2}$

$= \sqrt{5^2 + 1^2}$

$= \sqrt{26}$

d of AC $= \sqrt{(1+3)^2 + (-2-2)^2}$

$= \sqrt{4^2 + (-4)^2}$

$= \sqrt{32}$

d of BC $= \sqrt{(1-2)^2 + (-2-3)^2}$

$= \sqrt{(-1)^2 + (-5)^2}$

$= \sqrt{26}$

$\therefore$ $\triangle ABC$ is iscoseles because $AB = BC = \sqrt{26}$

and $AC = \sqrt{32}$

Annotations

Applies distance formula to sides of a triangle defined by coordinate pairs to prove the triangle is isosceles.

Algebra: Coordinate geometry

b) $(\text{height of } \Delta)^2 = \sqrt{26} - \frac{\sqrt{32}}{2}$

$= 26 - 8$

$\text{height}^2 = 18$

$\text{height} = \sqrt{18}$

$= 4.24$

$\therefore \text{A of ABC} = \frac{1}{2} b \times h$

$= \frac{1}{2} \times \left(\frac{\sqrt{32}}{2}\right) \times \sqrt{18}$

$= \frac{1}{2} \times 12$

$= 6$

Annotations

Uses the information already determined to calculate the area of the isosceles triangle.